Dawson College: Calculus II: 201-NYA-05 S18 November 3, 2011

Last Name: SOLUTIONS

First Name: __________

Student ID: __________

Test 2

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation.**

Question 1. Find the derivative of the following functions (do not simplify unless otherwise stated):

(a) *(2 marks)* $f(x) = 2\sec x + (x^3+4x)^{10}$

$$f'(x) = 2\sec x \tan x + 10(x^3+4x)^9 \cdot (3x^2+4)$$

(b) *(5 marks)* (Simplify your answer) $f(x) = \dfrac{(2x-1)^3}{(3x+5)^4}$

$$f'(x) = \frac{3(2x-1)^2 \cdot 2 \cdot (3x+5)^4 - (2x-1)^3 \cdot 4(3x+5)^3 \cdot 3}{[(3x+5)^4]^2}$$

$$= \frac{6(2x-1)^2(3x+5)^3[(3x+5) - 2(2x-1)]}{(3x+5)^8}$$

$$= \frac{6(2x-1)^2(3x+5)^3(-x+7)}{(3x+5)^8}$$

$$= \frac{-6(2x-1)^2(x-7)}{(3x+5)^5}$$

(c) *(4 marks)* $f(x) = (2x^2 - 1)\sqrt{5x^3 + x}$

$$f'(x) = (4x)\sqrt{5x^3+x} + (2x^2-1)\cdot \frac{1}{2}(5x^3+x)^{-1/2}\cdot(15x^2+1)$$

(d) *(5 marks)* $f(x) = \arcsin\sqrt{5x^2 - 4}$

$$f'(x) = \frac{1}{\sqrt{1-(\sqrt{5x^2-4})^2}}\cdot\frac{d}{dx}\left[\sqrt{5x^2-4}\right]$$

$$= \frac{1}{\sqrt{1-(5x^2-4)}}\cdot\frac{1}{2}(5x^2-4)^{-1/2}\cdot(10x)$$

Question 2. *(5 marks)* Let $f(x) = x\sin x$. Find the equation of the tangent line to the curve of $f(x)$ at the point $(\pi/2, \pi/2)$

$$f'(x) = 1 \cdot \sin x + x\cos x$$

$$m = f'\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} = 1 + \frac{\pi}{2}\cdot 0 = 1$$

$$y = mx + b$$

$$\frac{\pi}{2} = 1 \cdot \frac{\pi}{2} + b$$

$$0 = b$$

$\therefore$ $y = x$ IS THE EQUATION OF THE TANGENT LINE.

Question 3. *(6 marks)* Find the second derivative of the following function. Simplify your final answer.

$$f(x) = (3x^2 - 5)^{\frac{3}{2}}$$

$$f'(x) = \frac{3}{2}(3x^2-5)^{1/2} \cdot 6x = 9x(3x^2-5)^{1/2}$$

$$\therefore f''(x) = 9(3x^2-5)^{1/2} + 9x\left[\frac{1}{2}(3x^2-5)^{-1/2} \cdot 6x\right]$$

$$= 9(3x^2-5)^{1/2} + \frac{27x^2}{(3x^2-5)^{1/2}}$$

$$= \frac{9(3x^2-5) + 27x^2}{(3x^2-5)^{1/2}}$$

$$= \frac{54x^2 - 45}{(3x^2-5)^{1/2}}$$

$$= \frac{9(6x^2-5)}{(3x^2-5)^{1/2}}$$

Question 4. *(6 marks)* James is printing a book of his greatest math jokes from his classes. The weekly total cost of producing these books is given by

$$C(x) = 0.0000001x^3 - 0.04x^2 + 10x + 100$$

Believe it or not, the demand equation for these books is given by

$$p = -0.005x + 20$$

where x is the quatitiy demanded and p is the price per unit in dollars.
(a) Find the revenue function $R(x)$ and the profit function $P(x)$.
(b) Find the marginal profit function $P'(x)$.
(c) Assuming James is selling 200 books, use the marginal profit function to determine if he should try to sell 201 books. Why or why not?

a) $R(x) = xp = x(-0.005x + 20) = -0.005x^2 + 20x$

$P(x) = R(x) - C(x) = \underbrace{-[0.0000001x^3 - 0.04x^2 + 10x + 100]}_{-C(x)} + \underbrace{[-0.005x^2 + 20x]}_{+R(x)}$

$= -0.0000001x^3 + 0.035x^2 + 10x - 100$

b) $P'(x) = -0.0000003x^2 + 0.07x + 10$

c) $P'(200) = -0.0000003(200)^2 + 0.07(200) + 10$

$= -0.012 + 14 + 10$

$= 23.988$

YES HE SHOULD SELL 201 BOOKS SINCE HE WILL MAKE A PROFIT OF \$23.99 (APPROXIMATELY) ON THE 201ST BOOK.

Question 5. *(6 marks)* Using the demand function from Question 4:

$$p = -0.005x + 20$$

(a) Calculate the elasticity of demand function $E(p)$.
(b) Calculate the elasticity of demand when the price do the book is \$15. What should be done to the price to increase revenue?
(c) At what price is the demand for the book unitary?

a) $p = -0.005x + 20$

$0.005x = 200 - p$

$x = 4000 - 200p = f(p) \Rightarrow f'(p) = -200$

$\therefore E(p) = -\dfrac{p\,f'(p)}{f(p)} = -\dfrac{p(-200)}{4000-200p} = \dfrac{200p}{4000-200p} = \dfrac{200p}{200(20-p)} = \dfrac{p}{20-p}$

b) $E(15) = \dfrac{15}{20-15} = \dfrac{15}{5} = 3 > 1$ $\therefore$ DEMAND IS ELASTIC

$\therefore$ PRICE SHOULD BE LOWERED SLIGHTLY TO INCREASE REVENUE.

c) $E(p) = \dfrac{p}{20-p} = 1$

$\Rightarrow p = 20 - p$

$2p = 20$

$p = 10$

$\therefore$ DEMAND IS ELASTIC WHEN PRICE IS \$10.

Question 6. (4 marks) Find $f'(2)$ given that $g(2) = 1$, $h(2) = -1$, $g'(2) = 2$, $h'(2) = 3$ and

$$f(x) = \frac{x \cdot h(x)}{g(x)}$$

$$f'(x) = \frac{\frac{d}{dx}[x \cdot h(x)] \cdot g(x) - xh(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$= \frac{[1 \cdot h(x) + x \cdot h'(x)] \cdot g(x) - xh(x) \cdot g'(x)}{[g(x)]^2}$$

$$f'(2) = \frac{[h(2) + 2 \cdot h'(2)] \cdot g(2) - 2 \cdot h(2) \cdot g'(2)}{[g(2)]^2}$$

$$= \frac{[-1 + 2(3)] \cdot (1) - 2(-1)(2)}{(1)^2}$$

$$= \frac{5 + 4}{1}$$

$$= 9.$$

Bonus (3 marks) In class we discussed the relationship between elasticity and revenue. We determined that if demand is unitary, changing the price slightly will have little effect on the revenue. For most revenue functions, raising or lowering the price too much will result in a decrease in revenue. Draw the graph of a revenue function where this isn't the case. In particular, draw the graph of a revenue function where even though demand is unitary at a certain price, if price is increased enough revenue will be more than at the unitary price. Indicate on you graph the unitary price and the new price where revenue is greater.