P884 #16 SOLUTION

$$\vec{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\vec{i} + \frac{2t}{t^2+1}\vec{j} \Rightarrow \vec{r}\,'(t) = \frac{-4t}{(t^2+1)^2}\vec{i} + \frac{-2t^2+2}{(t^2+1)^2}\vec{j}$$

$$|\vec{r}\,'(t)| = \sqrt{\left[\frac{-4t}{(t^2+1)^2}\right]^2 + \left[\frac{-2t^2+2}{(t^2+1)^2}\right]^2} = \sqrt{\frac{4t^4+8t^2+4}{(t^2+1)^4}}$$

$$= \sqrt{\frac{4(t^2+1)^2}{(t^2+1)^4}} = \sqrt{\frac{4}{(t^2+1)^2}} = \frac{2}{t^2+1}$$

SINCE $(1,0) \Leftrightarrow t=0$

$$s(t) = \int_0^t |\vec{r}\,'(u)|\,du = \int_0^t \frac{2}{u^2+1}\,du = 2\arctan t.$$

$$\Rightarrow \arctan t = \tfrac{1}{2}s \Rightarrow t = \tan\left(\tfrac{1}{2}s\right)$$

$$\therefore\ r(t(s)) = \left[\frac{2}{\tan^2\left(\frac{1}{2}s\right)+1} - 1\right]\vec{i} + \frac{2\tan\left(\frac{1}{2}s\right)}{\tan^2\left(\frac{1}{2}s\right)+1}\vec{j}$$

$$= \frac{1-\tan^2\left(\frac{1}{2}s\right)}{1+\tan^2\left(\frac{1}{2}s\right)}\vec{i} + \frac{2\tan\left(\frac{1}{2}s\right)}{\sec^2\left(\frac{1}{2}s\right)}\vec{j}$$

$$= \frac{1-\tan^2\left(\frac{1}{2}s\right)}{\sec^2\left(\frac{1}{2}s\right)}\vec{i} + 2\tan\left(\tfrac{1}{2}s\right)\cos^2\left(\tfrac{1}{2}s\right)$$

$$= \left[\cos^2\left(\tfrac{1}{2}s\right) - \sin^2\left(\tfrac{1}{2}s\right)\right]\vec{i} + 2\sin\left(\tfrac{1}{2}s\right)\cos\left(\tfrac{1}{2}s\right)\vec{j}$$

$$= \cos s\,\vec{i} + \sin s\,\vec{j} = \langle \cos s,\ \sin s,\ 0\rangle$$