Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S1: Fall 2011

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(6 marks)* §8.2 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

10.

$$\sum_{n=1}^{\infty} \underbrace{\frac{n+1}{2n-3}}_{a_n} \qquad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{2n-3} = \frac{1}{2} \neq 0$$

diverges by n^{th} term divergence test

13.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + a_0 - a_0 = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n - a_0 = \frac{1}{1-1/3} - 1$$

since $|r_1| < 1$ where $r_1 = \frac{1}{3}$

$$= \frac{1}{\frac{2}{3}} - 1 = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n + a_0 - a_0 = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)^0 = \frac{1}{1-2/3} - 1$$

since $|r_2| < 1$ where $r_2 = \frac{2}{3}$

$$= \frac{1}{\frac{1}{3}} - 1 = 2$$

So $$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

$$= \frac{1}{2} + 2 = \frac{5}{2}$$

Question 2. *(4 marks)* §8.3 #26 Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \underbrace{\frac{n+5}{\sqrt[3]{n^7+n^2}}}_{a_n}$$

$\sum b_n$ where $b_n = \frac{n}{\sqrt[3]{n^7}} = \frac{1}{n^{4/3}}$ is a p-series where $p = \frac{4}{3}$ $\therefore$ convergent since $p > 1$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+5}{\sqrt[3]{n^7+n^2}}}{\frac{n}{\sqrt[3]{n^7}}} = \lim_{n\to\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}} \cdot \frac{\sqrt[3]{n^7}}{n}$$

$$= \lim_{n\to\infty} \frac{n+5}{n} \cdot \sqrt[3]{\frac{n^7}{n^7+n^2}}$$

$$= 1 \cdot 1$$

$$= 1 > 0 \text{ and finite}$$

$\therefore$ by limit comparison test $\sum a_n$ converges since $\sum b_n$ is a convergent series.