**Dawson College: Calculus II** (REGULAR): 201-NYB-05-S4: Fall 2011

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# Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(2 marks)* §8.1 #6 Find a formula for the general term $a_n$ of the sequence, assuming that the pattern of the first few terms continues.

$\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \cdots\right\}$ $\quad a_n = (-1)^n \frac{n}{(n+1)^2}$

**Question 2.** *(3 marks)* §8.1 #27 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{(n+2)!}{n!} = \frac{1\cdot 2\cdot 3\cdots (n-1)\,n(n+1)(n+2)}{1\cdot 2\cdot 3\cdots (n-1)\,n} = (n+1)(n+2)$$

$\therefore \; a_n \to \infty$ as $n \to \infty$

**Question 3.** *(5 marks)* §8.2 #19 Determine whether the series is convergent or divergent by expressing $S_n$ as a telescoping sum. If it is convergent find its sum.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$\frac{2}{(n-1)(n+1)} = \frac{A}{n-1} + \frac{B}{n+1}$$

$$2 = A(n+1) + B(n-1)$$

Let $n = -1$

$$2 = A(-1+1) + B(-1-1)$$

$$-1 = B$$

Let $n = 1$

$$2 = A(1+1) + B(1-1)$$

$$1 = A$$

$$S_n = a_2 + a_3 + a_4 + a_5 + a_6 + \cdots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1} - \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{1}{3} - \frac{1}{5}\right] + \left[\frac{1}{4} - \frac{1}{6}\right] + \left[\frac{1}{5} - \frac{1}{7}\right] + \cdots +$$

$$\left[\frac{1}{n-5} - \frac{1}{n-3}\right] + \left[\frac{1}{n-4} - \frac{1}{n-2}\right] + \left[\frac{1}{n-3} - \frac{1}{n-1}\right] + \left[\frac{1}{n-2} - \frac{1}{n}\right] + \left[\frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$= 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}\right]$$

$$= \frac{3}{2}$$

**Question 4.** *(5 marks)* Find the volume of the solid obtained when the region bounded by the graphs of $f(x) = \frac{4}{x}$, $y = 1$ and $g(x) = x$ is rotated about the line $y = -1$.

see test #3