Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S1: Fall 2011

Name: Y. Lamontagne
Student ID:

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.3 #19 Evaluate the integral.

$$\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}}\,dt = 6\left[\arcsin t\right]_{1/2}^{\sqrt{3}/2}$$

$$= 6\left[\arcsin\frac{\sqrt{3}}{2} - \arcsin\frac{1}{2}\right]$$

$$= 6\left[\frac{\pi}{3} - \frac{\pi}{6}\right]$$

$$= \pi$$

Question 2. *(5 marks)* §5.5 #62 If f is continuous and $\int_0^9 f(x)\,dx = 4$, find $\int_0^3 x f(x^2)\,dx$.

$$\int_0^3 x f(x^2)\,dx = \int_0^9 f(u)\,\frac{du}{2}$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u(3) = 3^2 = 9$

$u(0) = 0^2 = 0$

$$= \frac{1}{2}\int_0^9 f(u)\,du$$

$$= \frac{1}{2}(4)$$

$$= 2$$

Question 3. *(5 marks)* §6.1 #20 Evaluate the integral.

$$\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right)\, dx = \left[uv\right]_1^{\sqrt{3}} - \int_1^{\sqrt{3}} v\,du$$

$u = \arctan\left(\frac{1}{x}\right)$

$du = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot \frac{-1}{x^2}\, dx$

$du = \frac{-1}{x^2+1}\, dx$

$dv = dx$

$v = x$

$$= \left[x\arctan\left(\frac{1}{x}\right)\right]_1^{\sqrt{3}} - \int_1^{\sqrt{3}} x\,\frac{(-1)}{x^2+1}\,dx$$

$$= \left[\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan 1 + \int_1^{\sqrt{3}} \frac{x}{x^2+1}\,dx\right]$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u(\sqrt{3}) = (\sqrt{3})^2+1 = 4$

$u(1) = 1^2+1 = 2$

$$= \left[\frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \int_2^4 \frac{1}{u}\,\frac{du}{2}\right]$$

$$= \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \frac{1}{2}\left[\ln|u|\right]_2^4$$

$$= \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \frac{1}{2}\ln 4 - \frac{1}{2}\ln 2$$

$$= \frac{\sqrt{3}\pi}{6} - \frac{\pi}{4} + \ln\sqrt{2}$$