

## Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #29 Evaluate the integral.

$$\begin{aligned}
 \int_{\pi/6}^{\pi/2} \cot^2 x \, dx &= \int_{\pi/6}^{\pi/2} \csc^2 x - 1 \, dx = \left[ -\cot x - x \right]_{\pi/6}^{\pi/2} \\
 &= \left[ -\cot \frac{\pi}{2} - \frac{\pi}{2} \right] - \left[ -\cot \frac{\pi}{6} - \frac{\pi}{6} \right] \\
 &= \left[ -0 - \frac{\pi}{2} \right] - \left[ -\sqrt{3} - \frac{\pi}{6} \right] \\
 &= \sqrt{3} - \frac{2\pi}{6} \\
 &= \sqrt{3} - \frac{\pi}{3}
 \end{aligned}$$

Question 2. (5 marks) §6.2 #45 Evaluate the integral.

$$\begin{aligned}
 \int \frac{1}{\sqrt{x^2+16}} \, dx &= \int \frac{1}{\sqrt{(4\tan\theta)^2+16}} 4\sec^2\theta \, d\theta \\
 x &= 4\tan\theta \\
 dx &= 4\sec^2\theta \, d\theta \\
 \frac{\text{opp}}{\text{adj}} &= \frac{x}{4} = \tan\theta \\
 \begin{array}{c} \text{hyp} \\ \text{adj} \end{array} &= \frac{\sqrt{x^2+16}}{4} \\
 \therefore \sec\theta &= \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2+16}}{4} \\
 &= \int \frac{4\sec^2\theta}{\sqrt{16(\tan^2\theta+1)}} \, d\theta \\
 &= \int \frac{4\sec^2\theta}{\sqrt{16\sec^2\theta}} \, d\theta \\
 &= \int \sec\theta \, d\theta \\
 &= \ln|\sec\theta + \tan\theta| + C \\
 &= \ln\left| \frac{\sqrt{x^2+16}}{4} + \frac{x}{4} \right| + C
 \end{aligned}$$
