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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.3 #11 Evaluate the integral.

$$\int_2^3 \frac{1}{x^2-1}\,dx$$

$$\frac{1}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

$$1 = (x-1) + B(x+1)$$

Let $x=1$: $1 = (1-1) + B(1+1)$, $\frac{1}{2} = B$

Let $x=-1$: $1 = A(-1-1) + B(-1+1)$, $-\frac{1}{2} = A$

$$\int_2^3 \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\,dx$$

$$= \left[-\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1|\right]_2^3$$

$$= \left[-\frac{1}{2}\ln|3+1| + \frac{1}{2}\ln|3-1|\right] - \left[-\frac{1}{2}\ln|2+1| + \frac{1}{2}\ln|2-1|\right]$$

$$= -\frac{1}{2}\ln 4 + \frac{1}{2}\ln 2 + \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1$$

$$= \ln\sqrt{\frac{3}{2}}$$

Question 2. *(5 marks)* §6.3 #22 Evaluate the integral.

$$\int \frac{x^2-x+6}{x^3+3x}\,dx$$

$$\frac{x^2-x+6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$$

$$x^2-x+6 = A(x^2+3) + (Bx+C)x$$

Let $x=0$:

$0^2-0+6 = A(0^2+3) + (B(0)+C)(0)$

$6 = 3A$

$2 = A$

Let $x=1$:

$1^2-1+6 = A(1^2+3) + (B(1)+C(1))$

$6 = 4A + B + C$

$6 = 4(2) + B + C$

$-2 = B + C$ (1)

Let $x=-1$

$(-1)^2-(-1)+6 = A((-1)^2+3) + (B(-1)+C)(-1)$

$8 = 2(4) + B - C$

$0 = B - C$ (2)

(1) + (2)

$-2 = 2B$

$-1 = B$

$\therefore C = -1$

$$\int \frac{2}{x} + \frac{-x-1}{x^2+3}\,dx$$

$$= 2\ln|x| - \int \frac{x}{x^2+3}\,dx - \int \frac{1}{x^2+3}\,dx$$

$$= 2\ln|x| - \frac{1}{2}\ln|x^2+3| - \frac{1}{\sqrt{3}}\arctan\frac{x}{\sqrt{3}} + C$$

Question 3. *(5 marks)* §6.2 #44 Evaluate the integral.

$$\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx$$

$x = a\sec\theta$

$dx = a\sec\theta\tan\theta\,d\theta$

$$= \int \frac{\sqrt{(a\sec\theta)^2 - a^2}}{(a\sec\theta)^4}\, a\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{a^2\sec^2\theta - a^2}}{a^4\sec^4\theta}\, a\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{a^2(\sec^2\theta - 1)}}{a^4\sec^4\theta}\, a\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{a^2\tan^2\theta}\,\tan\theta}{a^3\sec^3\theta}\,d\theta$$

$$= \int \frac{a\tan\theta\tan\theta}{a^4\sec^3\theta}\,d\theta$$

$$= \frac{1}{a^2}\int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$

$$= \frac{1}{a^2}\int \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^3\theta}}\,d\theta$$

$$= \frac{1}{a^2}\int \sin^2\theta\cos\theta\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

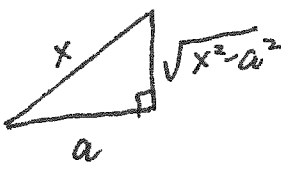
$$= \frac{1}{a^2}\int u^2\,du$$

$$= \frac{1}{a^2}\left[\frac{u^3}{3}\right] + C$$

$$= \frac{1}{a^2}\,\frac{\sin^3\theta}{3} + C$$

$$= \frac{\sin^3\theta}{3a^2} + C$$

$\frac{\text{hyp.}}{\text{adj.}} = \frac{x}{a} = \sec\theta$



So $\sin\theta = \frac{\sqrt{x^2-a^2}}{x}$

$$\therefore = \frac{\left(\frac{\sqrt{x^2-a^2}}{x}\right)^3}{3a^2} + C$$

$$= \frac{(\sqrt{x^2-a^2})^3}{3a^2x^3} + C$$