Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S1: Fall 2011

Name: Y. Lamontagne
Student ID:

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.6 #21 Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx = \int_{-\infty}^{0} \frac{x^2}{9+x^6}\,dx + \int_{0}^{\infty} \frac{x^2}{9+x^6}\,dx$$

$$= \lim_{a\to-\infty} \int_a^0 \frac{x^2}{9+(x^3)^2}\,dx + \lim_{b\to\infty} \int_0^b \frac{x^2}{9+(x^3)^2}\,d$$

$u = x^3$, $du = 3x^2\,dx$, $\frac{du}{3} = x^2\,dx$; $u(0)=0$, $u(a)=a^3$, $u(b)=b^3$

$$= \lim_{a\to-\infty} \int_{a^3}^{0} \frac{1}{3^2+u^2}\frac{du}{3} + \lim_{b\to\infty} \int_0^{b^3} \frac{1}{3^2+u^2}\frac{du}{3}$$

$$= \frac{1}{3}\left[\lim_{a\to-\infty}\left[\frac{1}{3}\arctan\frac{u}{3}\right]_{a^3}^{0} + \lim_{b\to\infty}\left[\frac{1}{3}\arctan\frac{u}{3}\right]_0^{b^3}\right]$$

$$= \frac{1}{3}\left[\lim_{a\to-\infty}\left[\frac{1}{3}\overbrace{\arctan 0}^{0} - \frac{1}{3}\overbrace{\arctan\frac{a^3}{3}}^{-\frac{\pi}{2}}\right] + \lim_{b\to\infty}\left[\frac{1}{3}\overbrace{\arctan\frac{b^3}{3}}^{\pi/2} - \frac{1}{3}\overbrace{\arctan\frac{0}{3}}^{0}\right]\right]$$

$$= \frac{1}{3}\left[-\left(\frac{1}{3}\right)\left(-\frac{\pi}{2}\right) + \frac{1}{3}\left(\frac{\pi}{2}\right)\right] = \frac{\pi}{9}$$

$\therefore$ converges to $\frac{\pi}{9}$

Question 2. *(5 marks)* §7.1 #8 Sketch the region enclosed by the given curves. Then find the area of the region.

$y = 1+\sqrt{x}, \quad y = 1+\frac{1}{3}x$

Lets find the intersection

$1+\sqrt{x} = 1+\frac{1}{3}x$

$\sqrt{x} = \frac{1}{3}x$

$x = \left(\frac{1}{3}x\right)^2$

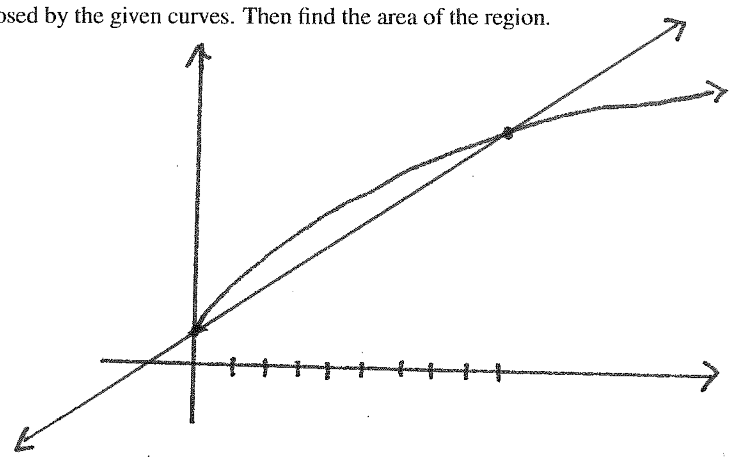
$x = \frac{x^2}{9}$

$9x = x^2$

$0 = x^2-9x$

$0 = x(x-9)$

$x=0 \quad x=9$



$$\text{Area} = \int_0^9 1+\sqrt{x} - \left(1+\frac{1}{3}x\right)dx$$

$$= \int_0^9 \sqrt{x} - \frac{1}{3}x\,dx$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{6}\right]_0^9$$

$$= \frac{2\,9^{3/2}}{3} - \frac{9^2}{6} = 18 - \frac{27}{2} = \frac{9}{2}$$

Question 3. *(5 marks)* If $f(0) = g(0) = 0$ and f'' and g'' are continuous, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$

see test #1.