Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S1: Fall 2011

Name: _____
Student ID: _____

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate using the definition of the definite integral

$$\int_1^2 -3x^2 + 2x - 1 \, dx.$$

Question 2. *(5 marks)* Evaluate the definite integral:

$$\int_{-\pi/4}^{\pi/6} |\tan\theta|\, d\theta$$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int 2(z^2+z)e^{4z^3+6z^2}\, dz$$

Question 4. *(5 marks)* Find the average value of the function

$$f(x) = (3x^2 + 1)\arctan x$$

on the interval $[0, 1]$.

Question 5. *(5 marks)* Evaluate the expression:

$$\frac{d}{dx}\left[\int_{2x}^{\csc 3x} u(\tan u)^u \, du\right]$$

Question 6. *(5 marks)* Evaluate the indefinite integral:

$$\int x \sec 2x \tan 2x \, dx$$

Question 7. *(5 marks)* Estimate the area under the graph of $f(x) = 4 - (x+2)^2$ from $x = -4$ to $x = -1$ using three rectangles and using midpoints. Sketch the curve and the approximating rectangles.

Question 8. *(5 marks)* Prove: If $f(x)$ is an odd integrable function on $[-a,a]$ then

$$\int_{-a}^{a} f(x)\,dx = 0$$

Question 9. *(5 marks)* If $f(0) = g(0) = 0$ and f'' and g'' are continuous, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx$$

Bonus Question. *(3 marks)*
Evaluate:

$$\lim_{h\to 0} \frac{\frac{\int_{\pi}^{x+h} \arctan(t)\, dt - \int_{\pi}^{x} \arctan(t)\, dt}{h}}{\cos h}$$