Name: _____

Student ID: _____

Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Evaluate the definite integral

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \sin^2(2x)\cos^2(2x)\,dx.$$

Question 2. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{x^2}{(9 - 4x^2)^{3/2}} \, dx$$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{x^2 - x + 6}{x^3 + 3x}\, dx$$

Question 4. *(5 marks)* Evaluate the improper integral or show it diverges:

$$\int_5^\infty \frac{1}{(x-1)\sqrt{x^2-2x-3}}\, dx$$

Question 5. *(5 marks)* Evaluate the improper integral or show it diverges:

$$\int_0^1 \frac{\ln x}{\sqrt[3]{x^2}} \, dx$$

Question 6. *(5 marks)* Sketch and find the area of the region(s) bounded by the graphs of $y = x$ and $y = \frac{1}{x^2}$ between $x = \frac{1}{2}$ and $x = 2$.

Question 7. *(5 marks)* Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $y = \sqrt{x+4}+1$, $y = 1$ and $y = x+3$ rotated about the line $y = 3$.

Question 8. *(5 marks)* Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $x = -y^2 + 2y$, $x = y$ rotated about the line $x = -1$.

Question 9. *(5 marks)* Find the length of the curve.

$$y = \ln(\cos x) \quad 0 \le x \le \frac{\pi}{3}$$

Bonus Question. *(3 marks)*
If $f(x)$ is a quartic function such that $f(0) = 4$, $f''(0) = 18$ and

$$\int \frac{f(x)}{x^2(x-1)^2(x^2+1)}\,dx$$

is a rational function, find the value of $f^{(4)}(x)$.