

Quiz 10

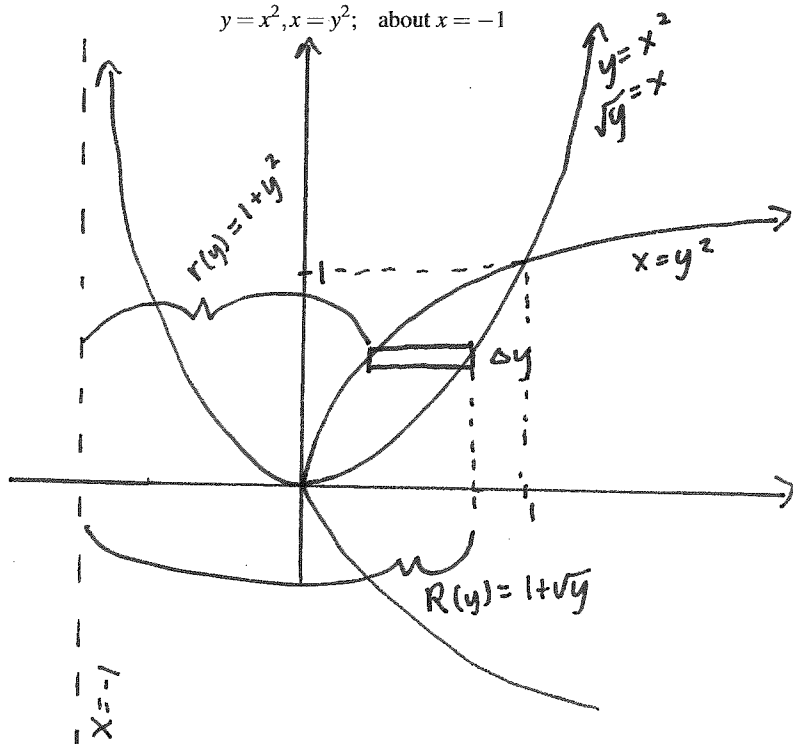
This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §7.4 #5 Find the length of the curve.

$$\begin{aligned}
 y &= \frac{x^5}{6} + \frac{1}{10x^3}, \quad 1 \leq x \leq 2 & y' &= \frac{5x^4}{6} - \frac{3}{10x^4} \\
 S &= \int_1^2 \sqrt{1 + (y')^2} \, dx & &= \left[\frac{x^5}{6} - \frac{1}{10x^3} \right]_1^2 \\
 &= \int_1^2 \sqrt{1 + \left(\frac{5x^4}{6} - \frac{3}{10x^4} \right)^2} \, dx & &= \frac{2^5}{6} - \frac{1}{10 \cdot 2^3} - \left[\frac{1}{6} - \frac{1}{10} \right] \\
 &= \int_1^2 \sqrt{1 + \frac{25x^8}{36} - \frac{1}{2} + \frac{9}{100x^8}} \, dx & &= \frac{1261}{240} \\
 &= \int_1^2 \sqrt{\frac{25x^8}{36} + \frac{1}{2} + \frac{9}{100x^8}} \, dx \\
 &= \int_1^2 \sqrt{\left(\frac{5x^4}{6} + \frac{3}{10x^4} \right)^2} \, dx \\
 &= \int_1^2 \left(\frac{5x^4}{6} + \frac{3}{10x^4} \right) \, dx
 \end{aligned}$$

Question 2. (5 marks) §7.2 #11 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region.

$y = x^2, x = y^2$; about $x = -1$



$$\begin{aligned}
 \Delta V &= \pi \left[(R(y))^2 - (r(y))^2 \right] \Delta y \\
 &= \pi \left[(1 + \sqrt{y})^2 - (1 + y^2)^2 \right] \Delta y \\
 V &= \int_0^1 \pi \left[(1 + \sqrt{y})^2 - (1 + y^2)^2 \right] dy \\
 &= \pi \int_0^1 \left[1 + 2\sqrt{y} + y - (1 + 2y^2 + y^4) \right] dy \\
 &= \pi \int_0^1 \left[2\sqrt{y} - 2y^2 + y - y^4 \right] dy \\
 &= \pi \left[\frac{2 \cdot 2}{3} y^{3/2} - \frac{2}{3} y^3 + \frac{y^2}{2} - \frac{y^5}{5} \right]_0^1 \\
 &= \pi \left[\frac{4}{3} - \frac{2}{3} + \frac{1}{2} - \frac{1}{5} \right] \\
 &= \frac{29\pi}{30}
 \end{aligned}$$