Dawson College: Calculus II (SCIENCE): 201-NYB-05-S1: Fall 2011

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.4 #34 Determine whether the series is absolutely convergent, conditionally convergent or divergent

$\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$ Lets apply the root test

$$\lim_{n\to\infty} \sqrt[n]{|a_n|}$$

$$= \lim_{n\to\infty}\left[\left|\frac{(-1)^n}{(\arctan n)^n}\right|\right]^{1/n}$$

$$= \lim_{n\to\infty}\left[\frac{1}{(\arctan n)^n}\right]^{1/n}$$

$$= \lim_{n\to\infty}\frac{1}{\arctan n} = \frac{1}{\frac{\pi}{2}}$$

$$= \frac{2}{\pi} < 1$$

$\therefore$ absolutely convergent.

Question 2. *(5 marks)* §8.5 #9 Find the radius of convergence and interval of convergence of the series.

$\sum_{n=1}^{\infty} \underbrace{\frac{(-2)^n x^n}{\sqrt[4]{n}}}_{a_n(x)}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\frac{(-2)^{n+1}x^{n+1}}{\sqrt[4]{n+1}}}{\frac{(-2)^n x^n}{\sqrt[4]{n}}}\right| = \lim_{n\to\infty}\left|\frac{(-2)^{n+1}x^{n+1}}{\sqrt[4]{n+1}}\cdot\frac{\sqrt[4]{n}}{(-2)^n x^n}\right|$$

$$= \lim_{n\to\infty}\left|(-2)x\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n+1}}\right|$$

$$= 2|x|\lim_{n\to\infty}\left|\frac{\sqrt[4]{n}}{\sqrt[4]{n+1}}\right|$$

$$= 2|x| < 1$$

So $2|x|<1$

$|x| < \frac{1}{2} = R$

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$

Lets check the endpoint

Let $x = -\frac{1}{2}$

$$\sum_{n=1}^{\infty}\frac{(-2)^n\left(-\frac{1}{2}\right)^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n}}$$

diverges since p-series where $p = \frac{1}{4} < 1$

Let $x = \frac{1}{2}$

$$\sum_{n=1}^{\infty}\frac{(-2)^n\left(\frac{1}{2}\right)^n}{\sqrt[4]{n}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[4]{n}}$$

Lets apply the alternating series test

Let $b_n = \frac{1}{\sqrt[4]{n}}$

- $b_{n+1} \overset{?}{\leq} b_n$

$$\frac{1}{\sqrt[4]{n+1}} \overset{?}{\leq} \frac{1}{\sqrt[4]{n}}$$

$$\sqrt[4]{n} \overset{?}{\leq} \sqrt[4]{n+1}$$

$$n \leq n+1 \checkmark$$

- $\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{\sqrt[4]{n}} = 0$

$\therefore$ Converges by the alternating series

$\therefore$ $R = \frac{1}{2}$ and interval of convergence is $\left(-\frac{1}{2}, \frac{1}{2}\right]$