

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #5 Evaluate the integral.

$$\begin{aligned}
 & \int_0^{\pi/2} \cos^2 x \, dx \\
 &= \int_0^{\pi/2} \frac{1 + \cos 2x}{2} \, dx \\
 &= \frac{1}{2} \int_0^{\pi/2} 1 + \cos 2x \, dx \\
 &= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]_0^{\pi/2} \\
 &= \frac{1}{2} \left[\left[\frac{\pi}{2} + \frac{\sin 2(\frac{\pi}{2})}{2} \right] - \left[0 + \frac{\sin 2(0)}{2} \right] \right] \\
 &= \frac{1}{2} \left[\left[\frac{\pi}{2} + \frac{\sin \pi}{2} \right] \right] \\
 &= \frac{\pi}{4}
 \end{aligned}$$

Question 2. (5 marks) §6.2 #31 Evaluate the integral.

$$\begin{aligned}
 \int \cot^3 x \csc^3 x \, dx &= \int \cot^2 x \csc^2 x \cot x \csc x \, dx \\
 &= \int (\csc^2 x - 1) \csc^2 x \cot x \csc x \, dx && \begin{aligned} u &= \csc x \\ du &= -\csc x \cot x \, dx \\ -du &= \csc x \cot x \, dx \end{aligned} \\
 &= \int (u^2 - 1) u^2 (-du) \\
 &= \int (1 - u^2) u^2 \, du \\
 &= \int u^2 - u^4 \, du \\
 &= \left[\frac{u^3}{3} - \frac{u^5}{5} \right] + C \quad \rightarrow \quad = \frac{\csc^3 x}{3} - \frac{\csc^5 x}{5} + C
 \end{aligned}$$