Dawson College: **Calculus II** (REGULAR): 201-NYB-05-S4: Fall 2011

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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.2 #47 Evaluate the integral.

$$\int \sqrt{1-4x^2}\,dx$$

$2x = \sin\theta$

$x = \frac{1}{2}\sin\theta$

$dx = \frac{1}{2}\cos\theta\,d\theta$

$$= \int \sqrt{1-(2x)^2}\,dx$$

$$= \int \sqrt{1-\sin^2\theta}\,\frac{1}{2}\cos\theta\,d\theta$$

$$= \int \sqrt{\cos^2\theta}\,\frac{1}{2}\cos\theta\,d\theta$$

$$= \frac{1}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \frac{1}{4}\int 1 + \cos 2\theta\,d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

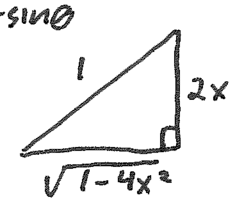
$$= \frac{1}{4}\theta + \frac{1}{8}2\sin\theta\cos\theta + C$$

$2x = \sin\theta$

$\arcsin(2x) = \arcsin \sin\theta$

$\arcsin(2x) = \theta$

$2x = \sin\theta$



$\therefore \cos\theta = \sqrt{1-4x^2}$

$$= \frac{1}{4}\arcsin(2x) + \frac{1}{8}2x\sqrt{1-4x^2} + C$$

$$= \frac{1}{4}\arcsin(2x) + \frac{x\sqrt{1-4x^2}}{4} + C$$

Question 2. *(5 marks)* §6.2 #46 Evaluate the integral.

$$\int \frac{t^5}{\sqrt{t^2+2}}\,dt$$

Solution #1

$$\int \frac{t^4\,t\,dt}{\sqrt{t^2+2}}$$

$u = t^2+2 \Leftrightarrow u-2 = t^2$

$du = 2t\,dt$

$\frac{du}{2} = t\,dt$

$$= \int \frac{(u-2)^2}{\sqrt{u}}\,\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{u^2-4u+4}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int u^{3/2} - 4u^{1/2} + 4u^{-1/2}\,du$$

$$= \frac{1}{2}\left[\frac{2u^{5/2}}{5} - \frac{2\cdot 4}{3}u^{3/2} + 8u^{1/2}\right] + C$$

$$= \frac{(t^2+2)^{5/2}}{5} - \frac{4}{3}(t^2+2)^{3/2} + 4\sqrt{t^2+2} + C$$

Solution #2

$$\int \frac{t^5}{\sqrt{t^2+2}}\,dt \qquad \begin{aligned} t &= \sqrt{2}\tan\theta \\ dt &= \sqrt{2}\sec^2\theta\,d\theta \end{aligned}$$

$$= \int \frac{(\sqrt{2}\tan\theta)^5}{\sqrt{(\sqrt{2}\tan\theta)^2+2}}\sqrt{2}\sec^2\theta\,d\theta$$

$$= \int \frac{(\sqrt{2})^5\tan^5\theta\,\sqrt{2}\sec^2\theta\,d\theta}{\sqrt{2\tan^2\theta+2}}$$

$$= \int \frac{(\sqrt{2})^6\tan^5\theta\sec^2\theta}{\sqrt{2(\tan^2\theta+1)}}\,d\theta$$

$$= \int \frac{(\sqrt{2})^6\tan^5\theta\sec^2\theta}{\sqrt{2\sec^2\theta}}\,d\theta$$

$$= \frac{(\sqrt{2})^6}{\sqrt{2}}\int \tan^5\theta\sec\theta\,d\theta$$

$$= (\sqrt{2})^5\int \tan^4\theta\tan\theta\sec\theta\,d\theta$$

$$= (\sqrt{2})^5\int (\sec^2\theta-1)^2\tan\theta\sec\theta\,d\theta$$

$u = \sec\theta$

$du = \tan\theta\sec\theta\,d\theta$

$$= (\sqrt{2})^5\int (u^2-1)^2\,du$$

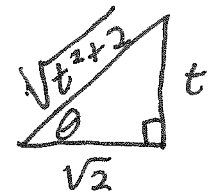
$$= (\sqrt{2})^5\int u^4-2u^2+1\,du$$

$$= (\sqrt{2})^5\left[\frac{u^5}{5}-\frac{2u^3}{3}+u\right]+C$$

$$= (\sqrt{2})^5\left[\frac{\sec^5\theta}{5}-\frac{2}{3}\sec^3\theta+\sec\theta\right]+C$$

$t = \sqrt{2}\tan\theta$

$\frac{t}{\sqrt{2}} = \tan\theta$



so $\sec\theta = \frac{\sqrt{t^2+2}}{\sqrt{2}}$

$$= (\sqrt{2})^5\left[\frac{\left(\frac{\sqrt{t^2+2}}{\sqrt{2}}\right)^5}{5}-\frac{2}{3}\left(\frac{\sqrt{t^2+2}}{\sqrt{2}}\right)^3+\frac{\sqrt{t^2+2}}{\sqrt{2}}\right]+C$$

$$= \left(\sqrt{t^2+2}\right)^5-\frac{4}{3}\left(\sqrt{t^2+2}\right)^3+4\sqrt{t^2+2}+C$$