Dawson College: Calculus II (REGULAR): 201-NYB-05-S4: Fall 2011

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Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.6 #31 Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$\int_0^2 z^2 \ln z\,dz$ infinite discontinuity at $x=0$

$$= \lim_{a\to 0^+} \int_a^2 z^2 \ln z\,dz \qquad u = \ln z \quad du = \frac{1}{z}dz \qquad v = \frac{z^3}{3} \quad dv = z^2 dz$$

$$= \lim_{a\to 0^+} \left[[uv]_a^2 - \int_a^2 v\,du \right]$$

$$= \lim_{a\to 0^+} \left[\left[\frac{z^3}{3}\ln z\right]_a^2 - \int_a^2 \frac{z^{\cancel{3}\,2}}{3}\frac{1}{\cancel{z}}dz \right]$$

$$= \lim_{a\to 0^+} \left[\frac{2^3}{3}\ln 2 - \frac{a^3}{3}\ln a - \left[\frac{z^3}{9}\right]_a^2 \right]$$

I.F. $0\cdot-\infty$

$$= \lim_{a\to 0^+} \left[\frac{8}{3}\ln 2 - \frac{\ln a}{3a^{-3}} - \frac{2^3}{9} + \frac{\cancel{a^3}}{9} \right] \quad (\to 0)$$

I.F. $\frac{-\infty}{\infty}$

$$= \frac{8}{3}\ln 2 - \frac{8}{9} - \lim_{a\to 0^+} \frac{1/a}{-9a^{-4}} \text{ by } \hat{H}$$

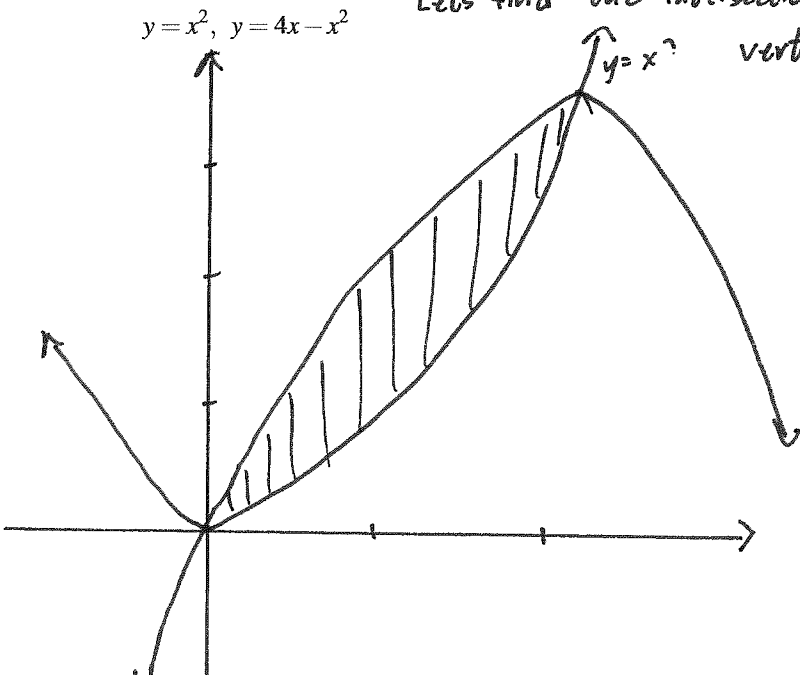
$$= \frac{8}{3}\ln 2 - \frac{8}{9} + \frac{1}{9}\lim_{a\to 0^+} \frac{a^{\cancel{4}\,3}}{\cancel{a}} = \frac{8}{3}\ln 2 - \frac{8}{9}$$

Question 2. *(5 marks)* §7.1 #10 Sketch the region enclosed by the given curves. Then find the area of the region.

$y = x^2$, $y = 4x - x^2$

Lets find the intersection of the two curves

$$x^2 = 4x - x^2$$
$$0 = 4x - 2x^2$$
$$0 = 2x(2-x)$$
$$x = 0 \qquad x = 2$$



vertex of $y = 4x - x^2$

$$y = -[x^2 - 4x]$$
$$= -[x^2 - 4x + 4 - 4]$$
$$= -[(x-2)^2 - 4]$$
$$= -(x-2)^2 + 4$$

$$A = \int_0^2 4x - x^2 - x^2\,dx$$

$$= \int_0^2 4x - 2x^2\,dx$$

$$= \left[\frac{4x^{\cancel{2}}}{\cancel{2}} - \frac{2x^3}{3}\right]_0^2$$

$$= \frac{4\,2^{\cancel{2}}}{\cancel{2}} - \frac{2\,2^3}{3} = 8 - \frac{16}{3} = \frac{8}{3}$$