Dawson College: **Calculus II** (REGULAR): 201-NYB-05-S4: Fall 2011

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum\limits_{i=1}^{n} c = cn$ where c is a constant $\quad \sum\limits_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum\limits_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum\limits_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate using the definition of the definite integral

$$\int_1^3 x^2 - 4x - 2\, dx.$$

Question 2. *(1 mark each)* Evaluate the following:

a.

$$\int_{\sqrt{2}}^{\sqrt{2}} x^2 + \frac{1}{1+x^2}\,dx.$$

b.

$$\int \sec x\,dx$$

c.

$$\int \tan x\,dx$$

d.

$$\int \frac{1}{\sqrt[4]{x^3}}\,dx$$

e. True or False: If $f(x)$ is continuous on $[a,b]$, then

$$\int_a^b kf(k)\,dk = k\int_a^b f(k)\,dk$$

Question 3. *(5 marks)* Evaluate the definite integral:

$$\int_{\pi/4}^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$$

Question 4. *(5 marks)* Evaluate the definite integral:

$$\int_1^2 \frac{z^2+1}{\sqrt[3]{z}}\, dz$$

Question 5. *(5 marks)* Find the average value of the function

$$f(x) = \frac{1}{x\sqrt{x^2-4}}$$

on the interval $[2\sqrt{2}, 4]$.

Question 6. *(5 marks)* Evaluate the expression:

$$\frac{d}{dx}\left[\int_{2x}^{\cot 3x} (\cos t)^t \, dt\right]$$

Question 7. *(5 marks)* If $\int_1^{\sqrt{3}} f(x)\, dx = 22$ then evaluate

$$\int_{\sqrt{3}}^{1} 24x^3 + 3f(x)\, dx$$

Question 8. *(5 marks)* Evaluate the integral by interpreting it in terms of areas.

$$\int_0^5 |x-2|+2\,dx$$

Question 9. *(5 marks)* Estimate the area under the graph of $f(x)=4-(x-1)^2$ from $x=-1$ to $x=2$ using three rectangles and using the midpoints. Sketch the curve and the approximating rectangles.

Question 10. *(5 marks)* Find the value(s) of c such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

for $f(x) = (x-3)^2$ on the interval $[2,5]$. Sketch the curve, indicate the c value(s).

Bonus Question. *(3 marks)*
Prove: If f and $g(x)$ is a continuous on [a,b] and c is a constant, then

$$\int_a^b f(x) + cg(x)\,dx = \int_a^b f(x)\,dx + c\int_a^b g(x)\,dx$$