Dawson College: **Calculus II** (REGULAR): 201-NYB-05-S4: Fall 2011

Name: Y. Lamontagne
Student ID:

Test 2

This test is graded out of 44 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Evaluate the integral.

$$\int x\sin(x^2)e^{\cos x^2}\,dx = \int e^u \frac{-du}{2} = -\frac{1}{2}\int e^u\,du$$

$u = \cos x^2$

$du = -\sin x^2 \cdot 2x$

$\frac{-du}{2} = x\sin(x^2)dx$

$$= -\frac{1}{2}e^u + C$$

$$= -\frac{1}{2}e^{\cos x^2} + C$$

Question 2. *(5 marks)* Evaluate the integral.

$$\int_{\sqrt[3]{5\pi/6}}^{\sqrt[3]{3\pi/4}} x^2\sec(\pi - x^3)\tan(\pi - x^3)\,dx = \int_{\pi/6}^{\pi/4} \sec u \tan u \frac{-du}{3}$$

$u = \pi - x^3$

$du = -3x^2dx \iff \frac{-du}{3} = x^2dx$

$$= -\frac{1}{3}\int_{\pi/6}^{\pi/4}\sec u\tan u\,du$$

$u\left(\sqrt[3]{3\pi/4}\right) = \pi - \left(\sqrt[3]{3\pi/4}\right)^3$

$= \pi - \frac{3\pi}{4}$

$= \frac{\pi}{4}$

$$= -\frac{1}{3}\left[\sec u\right]_{\pi/6}^{\pi/4}$$

$$= -\frac{1}{3}\left[\sec\frac{\pi}{4} - \sec\frac{\pi}{6}\right]$$

$$= -\frac{1}{3}\left[\sqrt{2} - \frac{2}{\sqrt{3}}\right]$$

$u\left(\sqrt[3]{5\pi/6}\right) = \pi - \left(\sqrt[3]{5\pi/6}\right)^3$

$= \pi - \frac{5\pi}{6}$

$= \frac{\pi}{6}$

Question 3. *(5 marks)* Evaluate the integral.

$$\int x^2 e^{3x}\,dx = uv - \int v\,du$$

$u = x^2$ $\quad du = 2x\,dx$

$v = \frac{e^{3x}}{3}$ $\quad dv = e^{3x}\,dx$

$$= \frac{x^2 e^{3x}}{3} - \int \frac{2xe^{3x}}{3}\,dx$$

$$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\int xe^{3x}\,dx$$

$u = x$ $\quad du = dx$

$v = \frac{e^{3x}}{3}$ $\quad dv = e^{3x}\,dx$

$$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[uv - \int v\,du\right]$$

$$= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx\right]$$

$$= \frac{x^2 e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2}{9}\int e^{3x}\,dx = \frac{x^2 e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + C$$

Question 4. *(5 marks)* Evaluate the integral.

$$\int_0^1 \arctan x\,dx$$

$u = \arctan x$ $\quad du = \frac{1}{1+x^2}\,dx$

$v = x$ $\quad dv = dx$

$$= \Big[uv\Big]_0^1 - \int_0^1 v\,du$$

$$= \Big[x\arctan x\Big]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \arctan 1 - 0\arctan 0 - \int_0^1 \frac{x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u(1) = 1+1^2 = 2$

$u(0) = 1+0^2 = 1$

$$= \frac{\pi}{4} - \int_1^2 \frac{1}{u}\frac{du}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$$= \frac{\pi}{4} - \frac{1}{2}\Big[\ln|u|\Big]_1^2$$

$$= \frac{\pi}{4} - \frac{1}{2}\Big[\ln 2 - \ln 1\Big]$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$

Question 5. *(5 marks)* Evaluate the integral.

$$\int_0^{\pi/4} \frac{\sin^3 x}{\sqrt{\cos x}}\,dx = \int_0^{\pi/4} \frac{\sin^2 x}{\sqrt{\cos x}} \sin x\,dx$$

$$= \int_0^{\pi/4} \frac{1-\cos^2 x}{\sqrt{\cos x}} \sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$u(\frac{\pi}{4}) = \cos \pi/4 = \frac{1}{\sqrt{2}}$

$u(0) = \cos 0 = 1$

$$= \int_1^{1/\sqrt{2}} \frac{1-u^2}{\sqrt{u}} -du$$

$$= -\int_1^{1/\sqrt{2}} \frac{1}{\sqrt{u}} - \frac{u^2}{\sqrt{u}}\,du$$

$$= -\int_1^{1/\sqrt{2}} u^{-1/2} - u^{3/2}\,du$$

$$= -\left[2u^{1/2} - \frac{2u^{5/2}}{5}\right]_1^{1/\sqrt{2}}$$

$$= -\left[2\sqrt{1/\sqrt{2}} - \frac{2}{5}\left(1/\sqrt{2}\right)^{5/2}\right] + \left[2 - \frac{2}{5}\right]$$

Question 6. *(5 marks)* Evaluate the integral.

$$\int \sec^4\theta \tan^{10}\theta\,d\theta = \int \tan^{10}\theta \sec^2\theta \sec^2\theta\,d\theta$$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$$= \int \tan^{10}\theta\,(1+\tan^2\theta)\sec^2\theta\,d\theta$$

$$= \int u^{10}(1+u^2)\,du$$

$$= \int u^{10} + u^{12}\,du$$

$$= \frac{u^{11}}{11} + \frac{u^{13}}{13} + C$$

$$= \frac{\tan^{11}\theta}{11} + \frac{\tan^{13}\theta}{13} + C$$

Question 7. *(5 marks)* Evaluate the integral.

$$\int \sqrt{4-9x^2}\,dx = \int \sqrt{2^2-(3x)^2}\,dx$$

$3x = 2\sin\theta$

$x = \frac{2}{3}\sin\theta$

$dx = \frac{2}{3}\cos\theta\, d\theta$

$$= \int \sqrt{2^2-(2\sin\theta)^2}\ \frac{2}{3}\cos\theta\, d\theta$$

$$= \int \sqrt{4-4\sin^2\theta}\ \frac{2}{3}\cos\theta\, d\theta$$

$$= \frac{2}{3}\int \sqrt{4(1-\sin^2\theta)}\ \cos\theta\, d\theta$$

$$= \frac{2}{3}\int \sqrt{4\cos^2\theta}\ \cos\theta\, d\theta$$

$$= \frac{2}{3}\int 2\cos\theta\cos\theta\, d\theta$$

$$= \frac{2}{3}\int 2\cos^2\theta\, d\theta$$

$$= \frac{4}{3}\int \cos^2\theta\, d\theta$$

$$= \frac{4}{3}\int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \frac{2}{3}\int 1+\cos 2\theta\, d\theta$$

$$= \frac{2}{3}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$= \frac{2}{3}\left[\arcsin\left(\frac{3x}{2}\right) + \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right] + C$$

$$= \frac{2}{3}\arcsin\left(\frac{3x}{2}\right) + \frac{\cancel{2}}{\cancel{3}}\,\frac{\cancel{3}x}{\cancel{2}}\,\frac{\sqrt{4-9x^2}}{2} + C$$

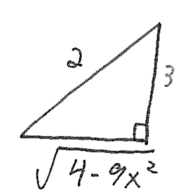
$$= \frac{2}{3}\arcsin\left(\frac{3x}{2}\right) + \frac{x\sqrt{4-9x^2}}{2} + C$$

note: $3x = 2\sin\theta$

$\frac{3x}{2} = \sin\theta$

$\theta = \arcsin\left(\frac{3x}{2}\right)$

and



So $\cos\theta = \frac{\sqrt{4-9x^2}}{2}$

Question 8. *(5 marks)* Evaluate the integral.

$$\int \frac{-x^2+3x+5}{x^3+5x}\,dx$$

$$\frac{-x^2+3x+5}{x(x^2+5)} = \frac{A}{x} + \frac{Bx+C}{x^2+5}$$

$$\frac{(-x^2+3x+5)\cancel{x(x^2+5)}}{\cancel{x(x^2+5)}} = \frac{A\cancel{x}(x^2+5)}{\cancel{x}} + \frac{(Bx+C)x\cancel{(x^2+5)}}{\cancel{x^2+5}}$$

$$-x^2+3x+5 = A(x^2+5) + (Bx+C)x$$

Let $x=0$

$$5 = 5A$$
$$1 = A$$

Let $x=1$

$$-1^2+3(1)+5 = A(1^2+5) + (B(1)+C)1$$
$$7 = 6A + B + C$$
$$7 = 6(1) + B + C$$
$$1 = B + C \quad (1)$$

Let $x=-1$

$$-(-1)^2+3(-1)+5 = A((-1)^2+5) + (B(-1)+C)(-1)$$
$$1 = 6A + B - C$$
$$1 = 6(1) + B - C$$
$$-5 = B - C \quad (2)$$

(1) + (2) $\quad -4 = 2B$
$$-2 = B$$

$\therefore \quad C = 3$

$$\int \frac{1}{x} + \frac{-2x+3}{x^2+5}\,dx = \int \frac{1}{x}\,dx - \int \frac{2x}{x^2+5}\,dx + \int \frac{3}{x^2+5}\,dx$$
$$= \ln|x| - \ln|x^2+5| + \frac{3}{\sqrt{5}}\arctan\frac{x}{\sqrt{5}} + C$$

Question 9. *(2 marks each)* True or False. Explain your reasoning.

a.
$$\int_{-\sqrt{2}}^{\sqrt{2}} x\tan^2 x \, dx = 0$$

true, since $f(x) = x\tan^2 x$ is an odd function

b. If $\int_0^4 f(x)\,dx = 3$ then $\int_0^2 xf(x^2)\,dx = \frac{3}{2}$.

true, since if we let $u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u(2) = 2^2 = 4$

$u(0) = 0^2 = 0$

so $\int_0^2 x f(x^2)dx = \int_0^4 f(u)\frac{du}{2}$

$= \frac{1}{2}\int_0^4 f(u)du$

$= \frac{1}{2}3 = \frac{3}{2}$

Bonus Question. *(3 marks)*
Evaluate the integral.

$$\int \csc^3 x \, dx = I$$

$u = \csc x \qquad du = -\csc x \cot x \, dx$

$v = -\cot x \qquad dv = \csc^2 x \, dx$

$I = \int \csc x \csc^2 x \, dx$

$I = uv - \int v\,du$

$I = -\cot x \csc x - \int -\cot x(-\csc x \cot x) dx$

$I = -\cot x \csc x - \int \cot^2 x \csc x \, dx$

$I = -\cot x \csc x - \int (\csc^2 x - 1)\csc x \, dx$

$I = -\cot x \csc x - \int \csc^3 x - \csc x dx$

$I = -\cot x \csc x - \underbrace{\int \csc^3 x \, dx}_{I} + \int \csc x \, dx$

$2I = -\cot x \csc x - \ln|\csc x + \cot x| + C$

$I = \frac{1}{2}\left[-\cot x \csc x - \ln|\csc x + \cot x|\right] + C$