

Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

## Test 3

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) Find the limit.

$$\lim_{x \rightarrow 1} \frac{3e^{1-x} - \ln x + \sin(4x-4) - 3}{x^2 - 2x + 1}$$

**Question 2.** (5 marks) Find the limit.

$$\lim_{x \rightarrow 0^+} (\cos x)^{1/x^2}$$

**Question 3.** (1 mark each)

a. (1 mark) True or False: Suppose that  $f(x)$  is continuous on  $(-\infty, \infty)$  then

$$\int_{-\infty}^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_{-t}^t f(x) dx$$

b. (2 marks) True or False, Justify.

$$\lim_{x \rightarrow 0^+} \sin x \ln x = \lim_{x \rightarrow 0^+} \frac{\cos x}{x}$$

**Question 4.** (5 marks) Evaluate the improper integral or show it diverges:

$$\int_{-\infty}^{\infty} \frac{x^2}{9+x^6} dx$$

**Question 5.** (5 marks) Evaluate the improper integral or show it diverges:

$$\int_0^2 z^2 \ln z \, dz$$

**Question 6.** (5 marks) Sketch the region enclosed by the given curves. Then find the area of the region.

$$y = |x|, \quad y = 2 - x^2$$

**Question 7.** (5 marks) Find the arc length of the graph of the function

$$x = \frac{1}{3}\sqrt{y}(y-3)$$

over the interval  $[1, 9]$ .

**Question 8.** (5 marks) Find the volume of the solid obtained when the region bounded by the graphs of  $y = 4x - x^2$ ,  $y = x$  is rotated about the line  $x$ -axis.

**Question 9.** (5 marks) Find the volume of the solid obtained when the region bounded by the graphs of  $f(x) = \frac{4}{x}$ ,  $y = 1$  and  $g(x) = x$  is rotated about the line  $y = -1$ .

**Bonus Question.** (1.5 marks each)

Sketch and label distances of the solid obtained when the region bounded by the graphs of:

a.  $(x - 1)^2 + y^2 = 1$  is rotated about the line  $x = -1$ .

b.  $x^2 + \frac{(y-2)^2}{4} = 1$  is rotated about the line  $x$ -axis.