Dawson College: Introduction to Applied Mathematics: 201-912-DW: Fall 2012

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Test 1

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

Equation of the least squares line: $y = mx + b$

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$b = \frac{(\sum x^2)(\sum y) - (\sum xy)(\sum x)}{n\sum x^2 - (\sum x)^2}$$

Question 1. *(5 marks)* Simplify the following expressing your final answer with positive exponents only:

$$\frac{\left(-\frac{1}{3}x^{-1}y^3\right)^{-2}}{x^2y^{-3}z} \div \left(\frac{x^6z}{3(x^{-1}y^2)^0z^6}\right)^{-1}$$

$$= \frac{\left(\frac{-1}{3}\right)^{-2}(x^{-1})^{-2}(y^3)^{-2}}{x^2y^{-3}z} \div \frac{(x^6z)^{-1}}{(3(x^{-1}y^2)^0z^6)^{-1}}$$

$$= \frac{9x^2y^{-6}}{x^2y^{-3}z} \div \frac{3z^6}{x^6z}$$

$$= \frac{39x^2y^{-6}}{x^2y^{-3}z} \cdot \frac{x^6z}{3z^6}$$

$$= 3\frac{1}{y^3} \cdot \frac{x^6}{z^6}$$

$$= \frac{3x^6}{y^3z^6}$$

Question 2. Convert the following and *use the correct number of signigicant figures*

a. *(2 marks)* 12.2 kN·m bending moment to ft·lb.

b. 2.37 miles per minutes to

i. *(2 marks)* km/hr

ii. *(2 marks)* ft/sec

a) $12.2\ \text{kN·m} \cdot \frac{10^3}{1\text{k}} \cdot \frac{0.225\ \text{lb}}{1\text{N}} \cdot \frac{3.28\ \text{ft}}{1\text{m}} = 9.00 \times 10^3\ \text{ft·lb}$

bi) $2.37\ \frac{\text{mi}}{\text{min}} \cdot \frac{60\ \text{min}}{1\ \text{hr}} \cdot \frac{1.61\ \text{km}}{1\ \text{mi}} = 229\ \text{km/hr}$

bii) $2.37\ \frac{\text{mi}}{\text{min}} \cdot \frac{1\ \text{min}}{60\ \text{s}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} = 209\ \text{ft/sec}$

Question 3. Let $f(x) = \sqrt{3x-1}$, evaluate and simplify the following:

a. *(1 mark)* $f(\frac{1}{3})$

b. *(1 mark)* $3f(x)$

c. *(1 mark)* $f(x+1)$

d. *(1 mark)* $f(a+h)$

e. *(1 mark)* $f(a)+f(h)$

a) $f(\frac{1}{3}) = \sqrt{3(\frac{1}{3})-1} = \sqrt{1-1} = 0$

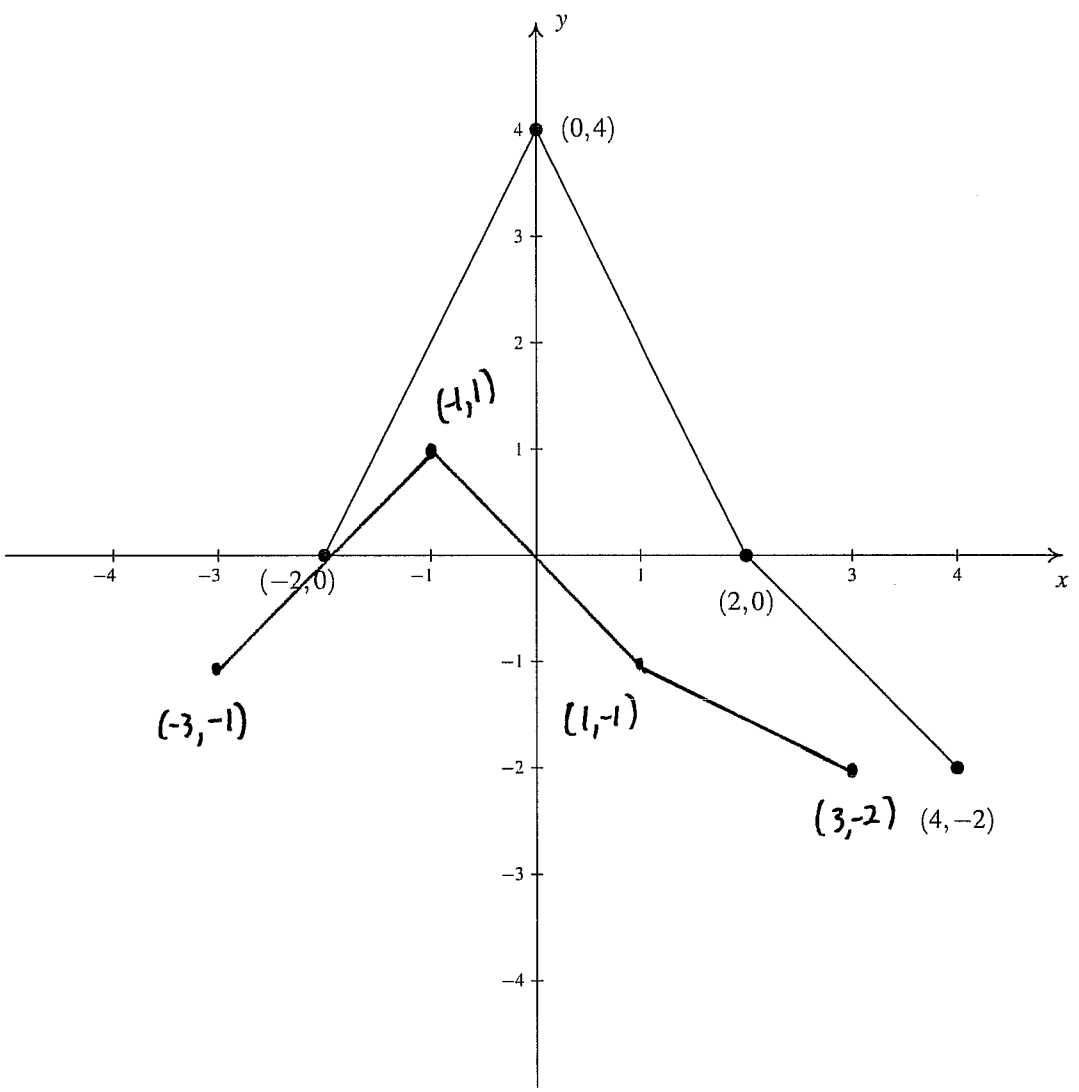
b) $3f(x) = 3\sqrt{3x-1}$

c) $f(x+1) = \sqrt{3(x+1)-1} = \sqrt{3x+3-1} = \sqrt{3x+2}$

d) $f(a+h) = \sqrt{3(a+h)-1} = \sqrt{3a+3h-1}$

e) $f(a) + f(x) = \sqrt{3a-1} + \sqrt{3x-1}$

Question 4. *(5 marks)* The complete graph of $y = f(x)$ is given below. Graph $y = \frac{1}{2}f(x+1) - 1$ on the set of axes.



Question 5. Let $f(x) = \sqrt{x+1}$, $g(x) = x^2 - 5$.

a. *(2 marks)* Simplify the expression $(f \circ g)(x)$.

b. *(1 mark)* Evaluate $(f \circ g)(0)$, if possible.

c. *(1 mark)* Evaluate $(f \circ g)(2)$, if possible.

d. *(2 marks)* State the domain of $f \circ g$.

a) $(f \circ g)(x) = f(g(x)) = f(x^2-5) = \sqrt{x^2-5+1} = \sqrt{x^2-4}$

b) $(f \circ g)(0) = \sqrt{0^2-4} = \sqrt{-4}$ not defined over $\mathbb{R}$

c) $(f \circ g)(2) = \sqrt{2^2-4} = \sqrt{4-4} = \sqrt{0} = 0$

d) $x^2-4 \geq 0$ $\therefore$ domain of $f \circ g$ is $(-\infty, -2] \cup [2, \infty)$

Question 6. *(5 marks)* Given

$$f(x) = \frac{2x-3}{4x+1}.$$

a. *(4 marks)* Find $f^{-1}(x)$.

b. *(1 mark)* State the domain of f and f^{-1}.

a) $y = \frac{2x-3}{4x+1}$

$x = \frac{2y-3}{4y+1}$

$x(4y+1) = 2y-3$

$4xy + x = 2y - 3$

$4xy - 2y = -3 - x$

$y(4x-2) = -3-x$

$y = \frac{-3-x}{4x-2}$

$\therefore f^{-1}(x) = \frac{-3-x}{4x-2}$

b) Domain of f: $4x+1 \neq 0$

$x \neq -1/4$

$\therefore \mathbb{R} \setminus \{-1/4\}$

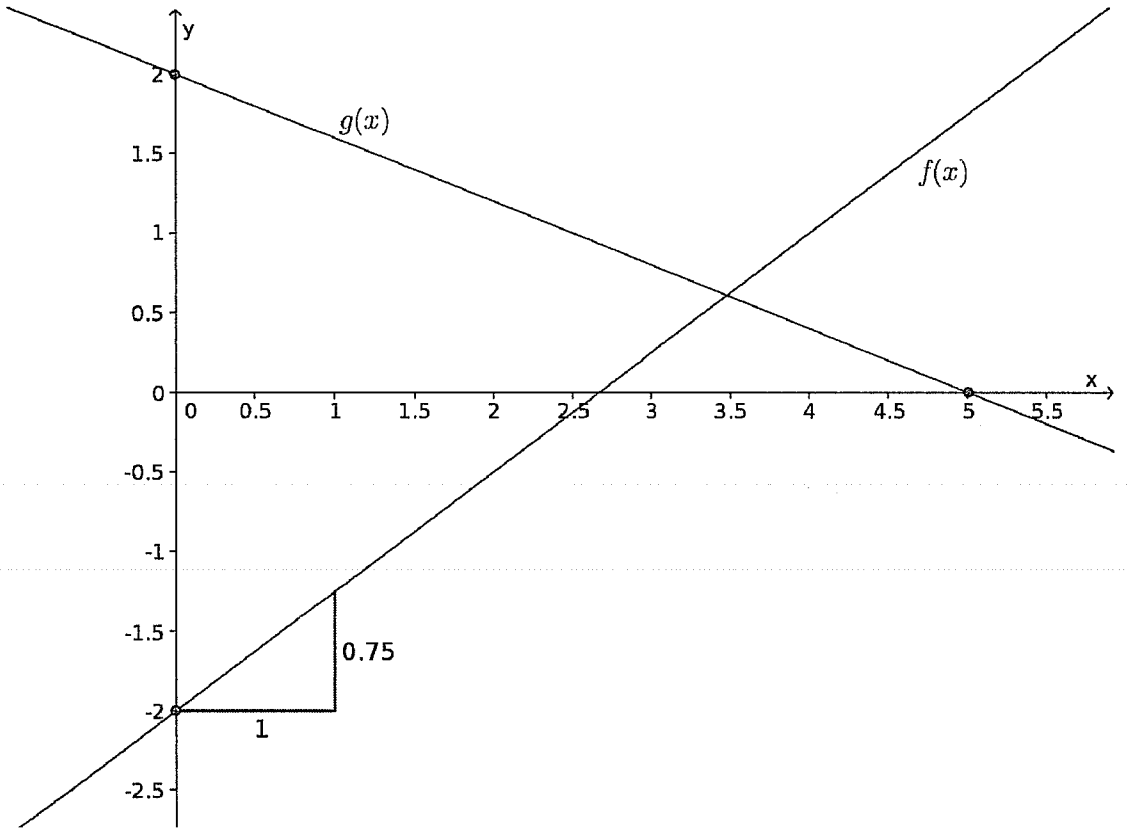
Domain of f^{-1}: $4x-2 \neq 0$

$x \neq 2/4$

$x \neq 1/2$

$\therefore \mathbb{R} \setminus \{1/2\}$

Question 7. *(6 marks)* Given the graph of $f(x)$ and $g(x)$. Determine the function $f(x)$ and $g(x)$. *Important: do not assume any unlabeled points on the graph.*



The points (0,2) and (5,0) lie on $g(x)$, so its slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{0-5} = \frac{2}{-5}$ and y-intercept 2. $\therefore$ $y = -\frac{2}{5}x + 2$

$g(x) = -\frac{2}{5}x + 2$

The point (0,-2) lies on $f(x)$ and its slope $m = \frac{\Delta y}{\Delta x} = \frac{0.75}{1} = \frac{3}{4}$ and y-intercept -2. $\therefore$ $y = \frac{3}{4}x - 2$

$f(x) = \frac{3}{4}x - 2$

Question 8. *(7 marks)* Sketch the graph of the function $f(x) = -4x^2 + 4x + 3$ by finding and using its vertex, x-intercepts and y-intercept. Also state the domain and range of the function.

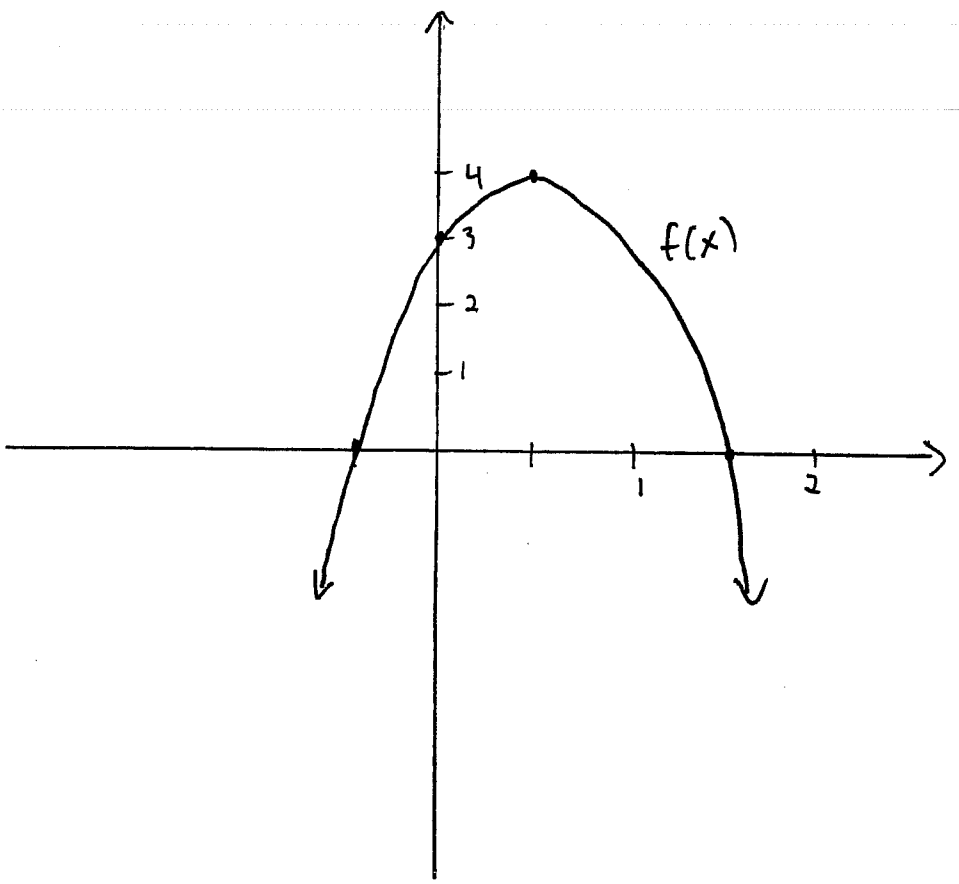
vertex:

$$f(x) = -4x^2 + 4x + 3$$
$$= -4\left[x^2 - x - \frac{3}{4}\right]$$
$$= -4\left[\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} - \frac{3}{4}\right]$$
$$= -4\left[(x - \tfrac{1}{2})^2 - 1\right]$$
$$= -4(x - \tfrac{1}{2})^2 + 4$$

$\therefore$ vertex at $(\frac{1}{2}, 4)$

x-intercept: $0 = f(x)$

$$0 = -4x^2 + 4x + 3$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(-4)(3)}}{2(-4)}$$
$$= \frac{-4 \pm \sqrt{16 + 48}}{-8}$$
$$= \frac{-4 \pm \sqrt{64}}{-8}$$
$$= \frac{-4 \pm 8}{-8}$$
$$= \frac{-12}{-8} \text{ or } \frac{4}{-8}$$
$$= \frac{3}{2} \text{ or } \frac{-1}{2}$$



y-intercept: $(0, f(0)) = (0, 3)$

domain: $\mathbb{R}$

range: $(-\infty, 4]$

Question 9. *(5 marks)* A contractor firm is investigating the relation, if any, between the estimated cost and the real cost of construction projects, based on the following data. Estimate the real cost for a contract whose estimated cost is 20 M$.

Estimated Cost (M$)	2	3	4	6	9	12	14	18
Real Cost (M$)	2	5	6	7	10	11	11	21

x, EC	y, RC	xy	x^2
2	2	4	4
3	5	15	9
4	6	24	16
6	7	42	36
9	10	90	81
12	11	132	144
14	11	154	196
18	21	378	324
68	73	839	810

$n = 8$

$$m = \frac{8(839) - (68)(73)}{8(810) - (68)^2} = 0.9418$$

$$b = \frac{(810)(73) - (839)(68)}{8(810) - (68)^2} = 1.1196$$

$\therefore y = mx + b$

$y = 0.9418x + 1.1196$

$\therefore y = 0.9418(20) + 1.1196$

$= 19.9556$

$= 20\ M\$$

Bonus Question. *(3 marks)* (From the 5th Dawson Mathematics Competition)
If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$ where a and b are real number, find a and b.

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$ and $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$

$f(bx + a) = x$

$a(bx + a) + b = x$

$abx + a^2 + b = x$

So $ab = 1 \Rightarrow a = \frac{1}{b}$ ①

$a^2 + b = 0$

② $b^2 + a = 0$

$f^{-1}(ax + b) = x$

$b(ax + b) + a = x$

$abx + b^2 + a = x$

sub ① into ② $b^2 + \frac{1}{b} = 0$

$b^3 = -1$

$b = -1$

$\therefore a = -1$