Dawson College: Introduction to Applied Mathematics: 201-912-DW: Fall 2012

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Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. In curing concrete the strength after t days of curing is given by the equation,

$$f = f_c(1 - e^{-kt}).$$

Where f_c is the ultimate strength.

a. *(3 marks)* If it takes 6 days for the concrete to get 65% of its ultimate strength, find k.

b. *(2 marks)* How long will it take for the concrete to get 92% of its ultimate strength.

a) $65\% = 100\%(1 - e^{-K(6)})$

$0.65 = 1 - e^{-6K}$

$e^{-6K} = 1 - 0.65$

$e^{-6K} = 0.35$

$\ln e^{-6K} = \ln 0.35$

$-6K = \ln 0.35$

$K = \frac{\ln 0.35}{-6}$

$K \doteq 0.1750$

b) $92\% = 100\%(1 - e^{-0.1750t})$

$0.92 = 1 - e^{-0.1750t}$

$e^{-0.1750t} = 1 - 0.92$

$e^{-0.1750t} = 0.08$

$\ln e^{-0.1750t} = \ln 0.08$

$-0.1750t = \ln 0.08$

$t = \frac{\ln 0.08}{-0.1750} \doteq 14.4$

∴ after 14 days the concrete will be at least 92% of its ultimate strength.

Question 2. *(5 marks)* Solve for x.

$\log_2 x + \log_2(x+2) = 3$

$\log_2 x(x+2) = 3$

$2^{\log_2 x(x+2)} = 2^3$

$x(x+2) = 8$

$x^2 + 2x = 8$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

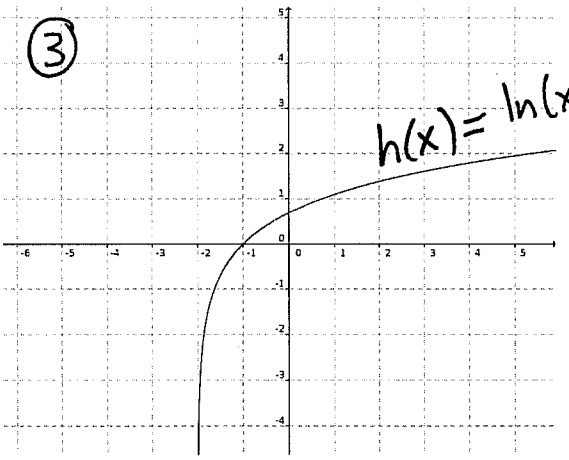
$x = -4$ $\quad$ $x = 2$

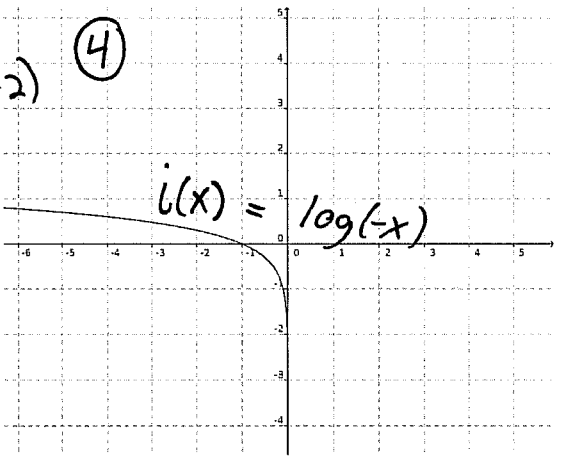
$x = -4$ is not a valid solution since $\log(-4)$ is not defined

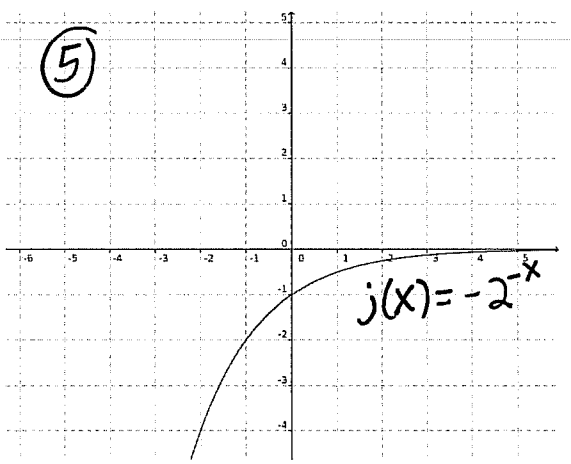
∴ $x = 2$

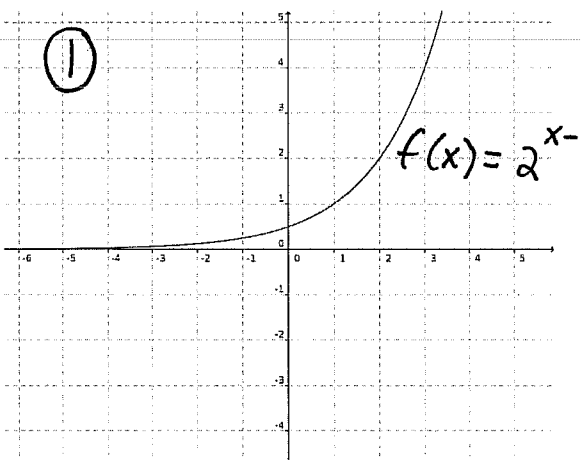
Question 3. *(5 marks)* Match the functions with their graphs. Write the function beside the correct graph.

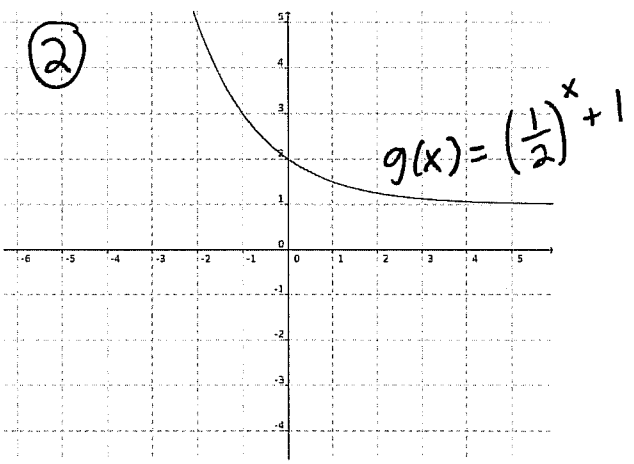
① $f(x) = 2^{x-1}$ ② $g(x) = \left(\frac{1}{2}\right)^x + 1$ ③ $h(x) = \ln(x+2)$ ④ $i(x) = \log(-x)$ ⑤ $j(x) = -2^{-x}$



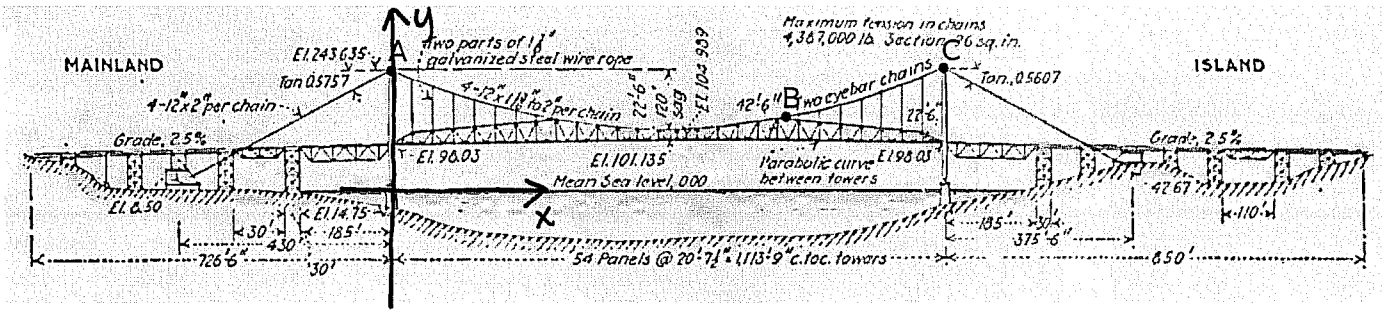








Question 4. *(5 marks)* The diagram[1] below describes the Hercilio Luz Bridge built in Florianopolis, Brazil. Find the equation of the parabola which approximate the suspended cable passing through A, B, C. Let the origin of the x, y axis be at water level of the pillar on the left. We can observe that $A = (0, 74)$, $B = (190, 44)$[2], $C = (340, 74)$ in meters.



$y = ax^2 + bx + c$

$(0, 74)$: $74 = a(0)^2 + b(0) + c$

$74 = c$

$(190, 44)$: $44 = a(190)^2 + b(190) + 74$

$44 = 36100a + 190b + 74$

$-30 = 36100a + 190b$

$-3 = 3610a + 19b$ ①

$(340, 74)$: $74 = a(340)^2 + b(340) + 74$

$0 = 115600a + 340b$

$0 = 340a + b$

$-340a = b$ ②

Sub ② into ①

$-3 = 3610a + 19(-340a)$

$-3 = -2850a$

$a = \frac{1}{950}$

$\therefore b = -340\left(\frac{1}{950}\right) = \frac{-34}{95}$

$\therefore y = \frac{1}{950}x^2 - \frac{34}{95}x + 74$

[1] published in the Engineering News-Record Vol. 91, No. 15, 1923.

[2] an approximation

Question 5. *(5 marks)* Solve for x_2 only by using Cramer's rule.

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 4 \\ 2x_1 - x_2 &= -2 \\ 4x_1 - 3x_3 &= 0 \end{aligned}$$

Let $|A| = \begin{vmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{vmatrix} = 1\begin{vmatrix} -1 & 0 \\ 0 & -3 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 0 \\ 4 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix}$

$= 3 + 3[-6] + 4 = -11$

$|A_2| = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & -3 \end{vmatrix} = 1\begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} - 4\begin{vmatrix} 2 & 0 \\ 4 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix}$

$= 6 - 4(-6) + 8$

$= 38$

$x_2 = \dfrac{|A_2|}{|A|} = \dfrac{38}{-11} = -\dfrac{38}{11}$

Question 6. *(5 marks)* A 60% sand mixture (MIX A) is to be mixed with a 35% sand mixture (MIX B) in order to make $10m^3$ of a 50% sand mixture. How much of MIX A and MIX B must be used in order to do this?

$A + B = 10 \longleftrightarrow B = 10 - A$ ①

$60\%A + 35\%B = 50\%10$

② $0.60A + 0.35B = 5$

sub ① into ②

$0.60A + 0.35(10 - A) = 5$

$0.60A + 3.5 - 0.35A = 5$

$0.25A = 1.5$

$A = 6\ m^3$

$\therefore A = 6m^3$ and $B = 4m^3$.

Question 7.

a. *(4 marks)* Find the center and the radius of the circle. *hint: complete the square for both variables.*

$$2x^2-4x+2y^2+8y=22$$

$x^2-2x+y^2+4y+11=0$

$(x^2-2x+1)-1+(y^2+4y+4)-4=11$

$(x-1)^2-1+(y+2)^2-4=11$

$(x-1)^2+(y+2)^2=16=4^2$

$\therefore r=4$ and center $(1,-2)$

b. *(4 marks)* Solve the non-linear system.

$$\begin{aligned} 2x^2-4x+2y^2+8y &= 22 \\ y-x+3 &= 0 \end{aligned}$$

c. *(3 marks)* Sketch the two equations from b.

b) $(x-1)^2+(y+2)^2=16$ ①

$y-x+3=0$ ②

$y=x-3$ ③

Sub ③ into ①

$(x-1)^2+(x-3+2)^2=16$

$(x-1)^2+(x-1)^2=16$

$2(x-1)^2=16$

$(x-1)^2=8$

$x-1=\pm\sqrt{8}$

$x=1\pm\sqrt{8}$

$x\doteq 3.828$ or -1.828

Let $x=1+\sqrt{8}$

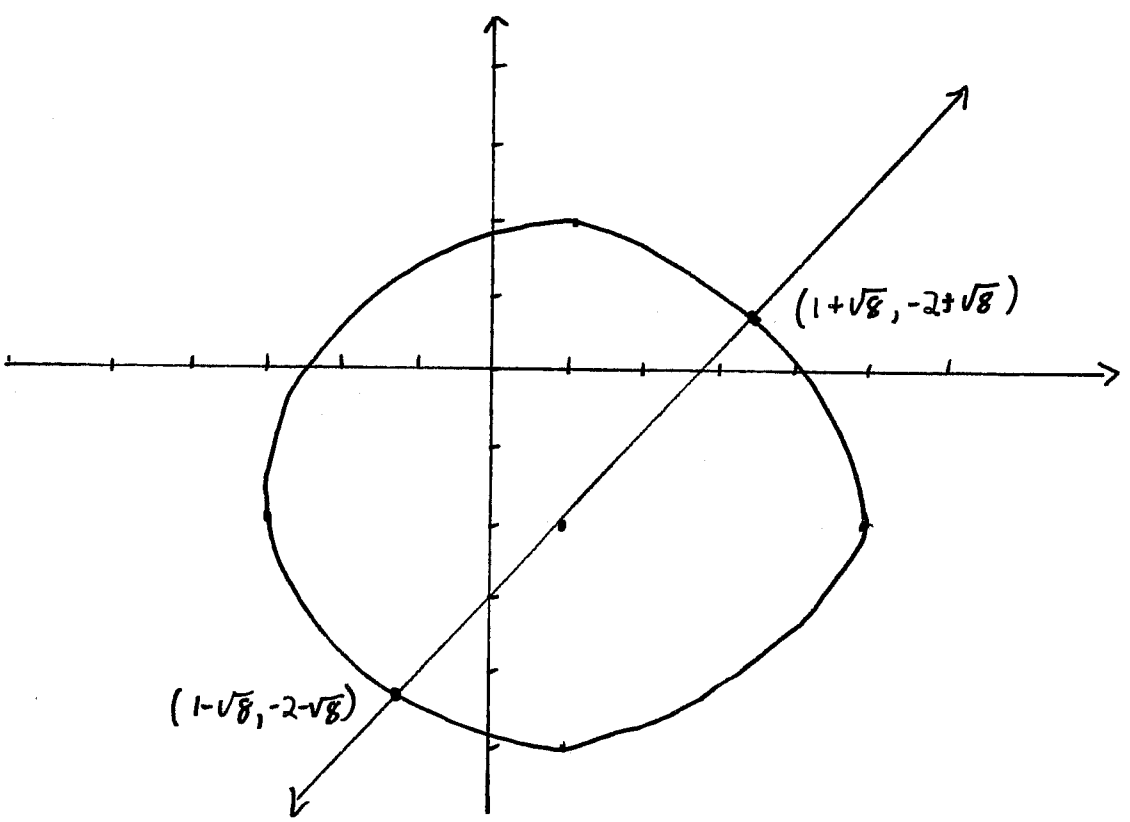
$y=1+\sqrt{8}-3$

$=-2+\sqrt{8}\doteq 0.828$

Let $x=1-\sqrt{8}$

$y=1-\sqrt{8}-3$

$=-2-\sqrt{8}\doteq -4.828$



Question 8.

a. Use the change of base formula and a calculator to evaluate the logarithm, correct to five decimal places:

i *(2 marks)*

$\log_7 \frac{1}{201} = \frac{\ln\left(\frac{1}{201}\right)}{\ln 7} \doteq -2.72536$

ii *(2 marks)*

$\log_6 216 = \frac{\ln 216}{\ln 6} = 3$

b. *(3 marks)* Expand the given logarithm and simplify.

$\log_5\left(\frac{25x^3z}{\sqrt{y}}\right)$

c. *(2 marks)* Use the properties of logarithms to write the expression as a single logarithm.

$\frac{1}{3}\log a + 4\log b - 2\log c$

$= \log \sqrt[3]{a} + \log b^4 - \log c^2$

$= \log\left(\frac{\sqrt[3]{a}\, b^4}{c^2}\right)$

b) $\log_5(25x^3z) - \log_5 \sqrt{y}$

$= \log_5 25 + \log_5 x^3 + \log_5 z - \frac{1}{2}\log_5 y$

$= 2 + 3\log_5 x + \log_5 z - \frac{1}{2}\log_5 y$

Bonus Question. *(3 marks)* Solve for x.

$2^x 3^x 5^x = e$

$\ln(2^x 3^x 5^x) = \ln e$

$\ln 2^x + \ln 3^x + \ln 5^x = 1$

$x\ln 2 + x\ln 3 + x\ln 5 = 1$

$x(\ln 2 + \ln 3 + \ln 5) = 1$

$x = \frac{1}{\ln 2 + \ln 3 + \ln 5}$

$x = 1/\ln 30$