Dawson College: **Introduction to Applied Mathematics**: 201-912-DW: Fall 2012

Name: _____
Student ID: _____

Test 3

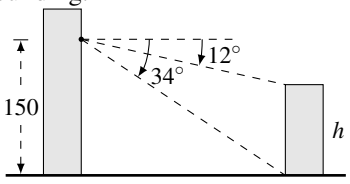
This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* One end of a rope is attached to the top of a pole 100.ft high. If the rope is 150.ft long, what is the maximum distance along the ground from the base of the pole to where the other end can be attached? You may assume that the pole is perpendicular to the ground.

Question 2. *(3 marks)* Find the values of the other five trigonometric functions of the acute angle θ $(0 < \theta < \frac{\pi}{2})$ given

$$\cot\theta = \frac{5}{9}$$

Question 3. *(4 marks)* From a position 150ft above the ground, an observer in a building measures angles of depression of 12° and 34° to the top and bottom, respectively, of a smaller building, as in the picture on the right. Use this to find the height h of the smaller building.



Question 4.

a. *(1 mark)* State in which quadrant or on which axis the angle 1059° and -512° lie.

b. *(1 mark)* In which quadrant(s) do sine and tangent have the opposite sign?

c. *(1 mark)* Find the reference angle for the angle 260°.

d. *(1 mark)* Convert the angle 120° to radians.

e. *(1 mark)* Convert the angle $\frac{5\pi}{6}$ to degrees.

f. *(4 marks)* Use $15^\circ = 45^\circ - 30^\circ$ to find the exact value of $\tan 15^\circ$. SHOW ALL YOUR WORK

Question 5. *(5 marks)* Find the amplitude, period, and phase shift *(displacement)*. Then graph one period of the given function.

$$y = \sin(2\pi x - \pi) + 1$$

Question 6. *(5 marks)* Solve the following equation in the interval $[0, 2\pi)$.

$$4\cos^2 t - 1 = 0$$

Question 7. *(5 marks)* Prove the given identity.

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\cot B - \cot A}{\cot B + \cot A}$$

Question 8. *(5 marks) (Use the correct number of significant figures)* Solve the triangle ΔABC where $A = 41^\circ$, $B = 45^\circ$, $c = 15$. Then find the area of the triangle.

Question 9. *(5 marks) (Use the correct number of significant figures)* Solve the triangle ΔABC where $A = 30.^\circ$, $b = 4.0$, $c = 5.0$. Then find the area of the triangle.

Question 10.

a. *(3 marks)* A circular arc of length 12 meters subtends a central angle of 24 degrees. Find the radius of the circle in meters.

b. *(3 marks)* Find the area of a sector whose angle is 117° in a circle of radius 3.5m.

Bonus Question. *(5 marks) (Similar to a question on the 6^{th} Dawson Math Competition)* If $f\left(\frac{\theta}{2}\right) = \sin\theta + \tan\theta$ for all real numbers, find $f(\theta)$ in terms of $\sin\theta$ and $\cos\theta$ only.

Formulae:

Equation of the least squares line: $y = mx + b$

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$b = \frac{\left(\sum x^2\right)(\sum y) - (\sum xy)(\sum x)}{n\sum x^2 - (\sum x)^2}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$