Dawson College: **Introduction to Applied Mathematics**: 201-912-DW: Fall 2012

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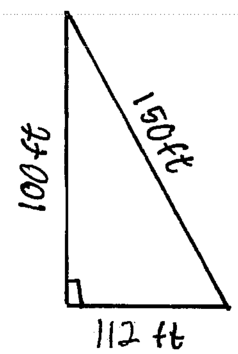
Student ID:

Test 3

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* One end of a rope is attached to the top of a pole 100ft high. If the rope is 150ft long, what is the maximum distance along the ground from the base of the pole to where the other end can be attached? You may assume that the pole is perpendicular to the ground.

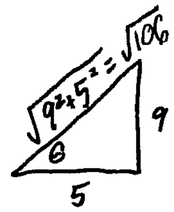


$$a^2 + b^2 = c^2$$
$$100^2 + b^2 = (150)^2$$
$$b^2 = 150^2 - 100^2$$
$$b = \sqrt{12500} \doteq 112\text{ ft}$$

Question 2. *(3 marks)* Find the values of the other five trigonometric functions of the acute angle θ $(0 < \theta < \frac{\pi}{2})$ given

$\cot\theta = \frac{5}{9}$

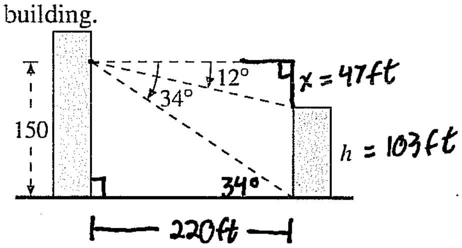
$\tan\theta = \frac{9}{5} = \frac{opp}{adj}$



$\sin\theta = \frac{9}{\sqrt{106}}$ $\quad \cos\theta = \frac{5}{\sqrt{106}}$

$\csc\theta = \frac{\sqrt{106}}{9}$ $\quad \sec\theta = \frac{\sqrt{106}}{5}$

Question 3. *(4 marks)* From a position 150ft above the ground, an observer in a building measures angles of depression of 12° and 34° to the top and bottom, respectively, of a smaller building, as in the picture on the right. Use this to find the height h of the smaller building.



$$\tan 34^\circ = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 34^\circ = \frac{150}{adj}$$
$$adj = \frac{150}{\tan 34^\circ} \doteq 222\text{ ft} = 220$$

$$\tan 12^\circ = \frac{x}{220}$$

$47\text{ft} = 46.7\text{ft} = 220\tan 12^\circ = x$

$\therefore h = 103\text{ft}$

$h = 1.0 \times 10^2\text{ ft}.$

Question 4.

a. *(1 mark)* State in which quadrant or on which axis the angle 1059° and −512° lie.

b. *(1 mark)* In which quadrant(s) do sine and tangent have the opposite sign?

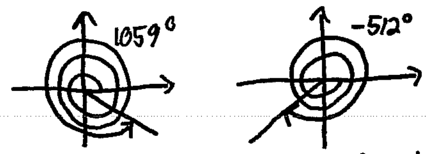
c. *(1 mark)* Find the reference angle for the angle 260°.

d. *(1 mark)* Convert the angle 120° to radians.

e. *(1 mark)* Convert the angle $\frac{5\pi}{6}$ to degrees.

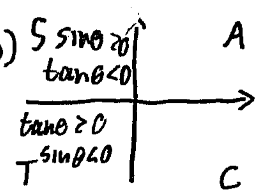
f. *(4 marks)* Use $15° = 45° - 30°$ to find the exact value of $\tan 15°$. SHOW ALL YOUR WORK

a)

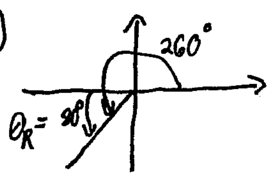


IV quadrant   III quadrant

b)



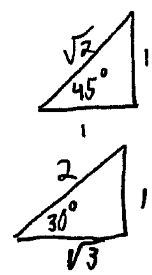
c)



d) $120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3}$ rad

e) $\frac{5\pi}{6}\left(\frac{180°}{\pi}\right) = 150°$

$$\tan(45 - 30°) = \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$



$$= \frac{\frac{1}{1} - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Question 5. *(5 marks)* Find the amplitude, period, and phase shift *(displacement)*. Then graph one period of the given function.

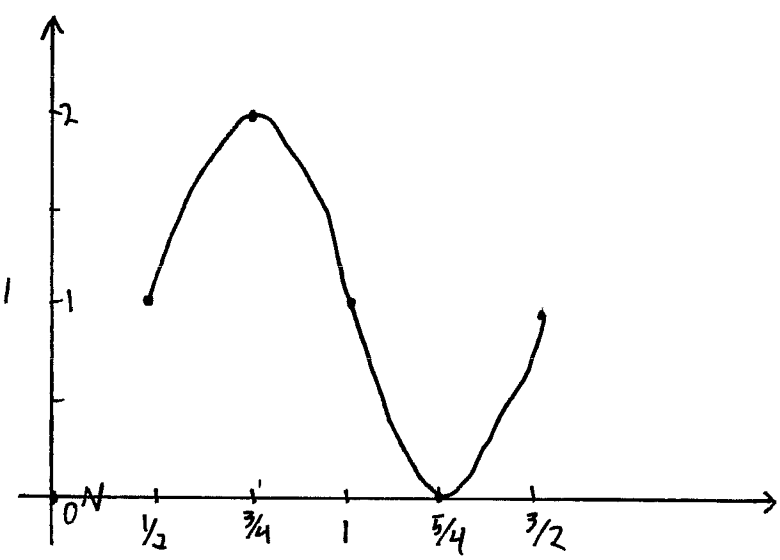
$y = \sin(2\pi x - \pi) + 1$

amplitude $= |a| = |1| = 1$

period $= \frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$

displacement $= \frac{-c}{b} = \frac{-(-\pi)}{2\pi} = \frac{1}{2}$

key values = displacement + mult. × $\frac{\text{period}}{4}$	$y = \sin(2\pi x - \pi) + 1$
$\frac{1}{2} + 0\left(\frac{1}{4}\right) = \frac{1}{2}$	$0 + 1 = 1$
$\frac{1}{2} + 1\left(\frac{1}{4}\right) = \frac{3}{4}$	$1 + 1 = 2$
$\frac{1}{2} + 2\left(\frac{1}{4}\right) = 1$	$0 + 1 = 1$
$\frac{1}{2} + 3\left(\frac{1}{4}\right) = \frac{5}{4}$	$-1 + 1 = 0$
$\frac{1}{2} + 4\left(\frac{1}{4}\right) = \frac{6}{4}$	$0 + 1 = 1$

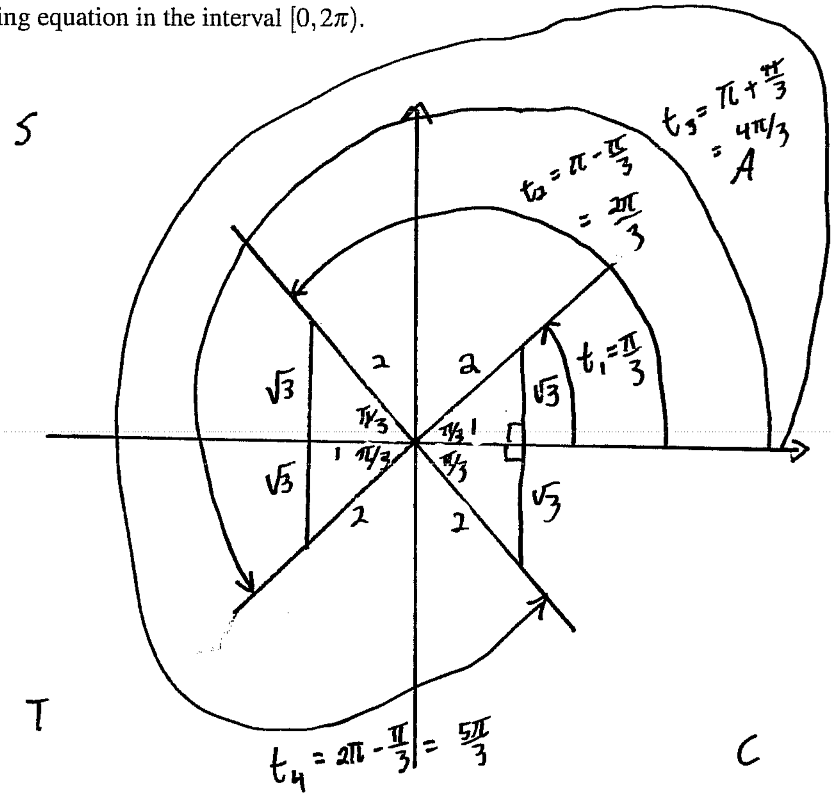


Question 6. *(5 marks)* Solve the following equation in the interval $[0, 2\pi)$.

$4\cos^2 t - 1 = 0$

$\cos^2 t = \frac{1}{4}$

$\cos t = \pm\frac{1}{2} = \frac{adj}{hyp}$



$\therefore t = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Question 7. *(5 marks)* Prove the given identity.

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\cot B - \cot A}{\cot B + \cot A}$$

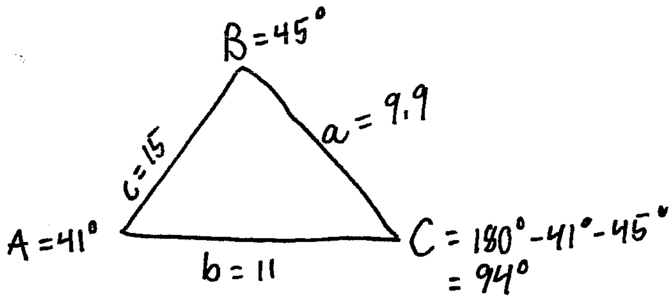
$$\frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} \stackrel{?}{=} \frac{\frac{\cos B}{\sin B} - \frac{\cos A}{\sin A}}{\frac{\cos B}{\sin B} + \frac{\cos A}{\sin A}}$$

$$" \stackrel{?}{=} \frac{\frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B}}{\frac{\cos B \sin A}{\sin A \sin B} + \frac{\cos A \sin B}{\sin A \sin B}}$$

$$" \stackrel{?}{=} \frac{\frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B}}{\frac{\cos B \sin A + \cos A \sin B}{\sin A \sin B}}$$

$$" = \frac{\sin A \cos B - \cos A \sin B}{\cos B \sin A + \cos A \sin B} \checkmark$$

Question 8. *(5 marks) (Use the correct number of significant figures)* Solve the triangle ΔABC where $A = 41°$, $B = 45°$, $c = 15$. Then find the area of the triangle.



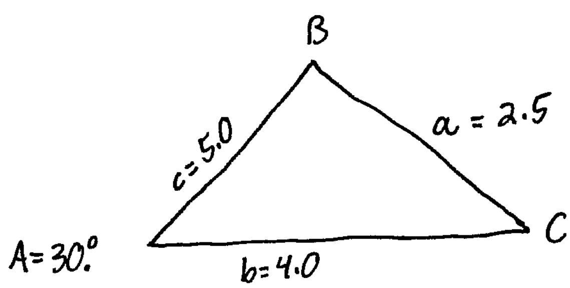
$$\text{Area} = \frac{c^2 \sin A \sin B}{2 \sin C}$$
$$= \frac{15^2 \sin 41° \sin 45°}{2 \sin 94°}$$
$$= 52.\ u^2$$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \qquad \frac{c}{\sin C} = \frac{b}{\sin B}$$

$$a = \frac{c \sin A}{\sin C} = \frac{15 \sin 41°}{\sin 94} = 9.9$$

$$b = \frac{c \sin B}{\sin C} = \frac{15 \sin 45°}{\sin 94°} = 11$$

Question 9. *(5 marks) (Use the correct number of significant figures)* Solve the triangle ΔABC where $A = 30.°$, $b = 4.0$, $c = 5.0$. Then find the area of the triangle.



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = (4.0)^2 + (5.0)^2 - 2(4.0)(5.0)\cos 30°$$
$$a^2 = 6.3589\ldots$$
$$a = 2.5$$

and

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$4.0^2 = 2.5^2 + 5.0^2 - 2(2.5)(5.0)\cos B$$
$$\cos B = \frac{4.0^2 - 2.5^2 - 5.0^2}{-2(2.5)(5.0)}$$
$$\cos B = 0.61$$
$$B = 52°$$

So $C = 180° - 30.° - 52° = 98.°$

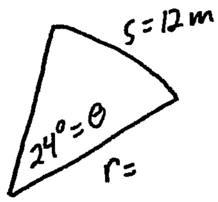
$$\text{Area} = \frac{1}{2} bc \sin A$$
$$= \frac{1}{2}(4.0)(5.0)\sin 30.°$$
$$= 5.0\ u^2$$

Question 10.

a. *(3 marks)* A circular arc of length 12 meters subtends a central angle of 24 degrees. Find the radius of the circle in meters.

b. *(3 marks)* Find the area of a sector whose angle is 117° in a circle of radius 3.5m.

a)



$\theta = 24° \frac{\pi}{180°}$

$= \frac{2\pi}{15}$

$\theta = \frac{s}{r}$

$\frac{2\pi}{15} = \frac{12}{r}$

$r = \frac{12(15)}{2\pi}$

$= \frac{6(15)}{\pi}$

$= 29m$

b) $A = \frac{1}{2}\theta r^2$

$\theta = 117°\left(\frac{\pi}{180°}\right) = \frac{117}{180}\pi$

$A = \frac{1}{2}\frac{117\pi}{180}(3.5)^2$

$= 13. m^2$

Bonus Question. *(5 marks) (Similar to a question on the 6th Dawson Math Competition)* If $f\left(\frac{\theta}{2}\right) = \sin\theta + \tan\theta$ for all real numbers, find $f(\theta)$ in terms of $\sin\theta$ and $\cos\theta$ only.

Let $\phi = \frac{\theta}{2}$

$2\phi = \theta$

So $f\left(\frac{2\phi}{2}\right) = \sin 2\phi + \tan 2\phi$

$f(\phi) = 2\sin\phi\cos\phi + \frac{2\tan\phi}{1-\tan^2\phi}$

So

$f(\theta) = 2\sin\theta\cos\theta + \frac{2\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$

$= 2\sin\theta\cos\theta + \frac{2\sin\theta\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}(\cos^2\theta-\sin^2\theta)}$

$\therefore f(\theta) = 2\sin\theta\cos\theta + \frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$