Question 1. *(10 marks)*
Simplify the given expressions. Express the results with positive exponents only.

(a) $\pi\left(\frac{r}{3}\right)^3\left(\frac{4}{3\pi r^2}\right)$

$$= \pi \cdot \frac{r^3}{27} \cdot \frac{4}{3\pi r^2} = \frac{4\pi r^3}{81\pi r^2} = \boxed{\frac{4\pi r}{81}}$$

(b) $\frac{(3^2t)^{-1}}{3t^{-1}} = \frac{t}{3(3^2t)} = \boxed{\frac{1}{27}}$

(c) $\left(\frac{4a^{\frac{5}{6}}b^{-\frac{1}{5}}}{a^{\frac{2}{3}}b^2}\right)^{-\frac{1}{2}} = \frac{4^{-1/2}a^{-5/12}b^{1/10}}{a^{-\frac{1}{3}}b^{-1}}$

$$= \frac{a^{1/3}b^{1/10}b}{4^{1/2}a^{5/12}} = \frac{a^{-1/12}b^{11/10}}{2} = \boxed{\frac{b^{11/10}}{2a^{1/12}}}$$

(d) $\left(\frac{3^{-1}a^{\frac{1}{2}}}{4^{-\frac{1}{2}}b}\right) \div \left(\frac{9^{\frac{1}{2}}a^{-\frac{1}{3}}}{2b^{-\frac{1}{4}}}\right)$

$$= \frac{4^{1/2}a^{1/2}}{3b} \cdot \frac{2b^{+1/4}}{9^{1/2}a^{-1/3}}$$ ← multiplication by reciprocal

$$= \frac{2a^{1/2}}{3b} \cdot \frac{2a^{1/3}}{3b^{1/4}}$$

$$= \boxed{\frac{4a^{5/6}}{9b^{5/4}}}$$

Question 2. *(6 marks)*
Simplify the given algebraic expressions.

(a) $-(3t-(7+2t-(5t-6)))$

$= -(3t-(7+2t-5t+6))$
$= -(3t-(-3t+13))$
$= -(6t-13)$
$= \boxed{-6t+13}$

(b) $5V^2-(6-(2V^2+3))$

$= 5V^2-(6-2V^2-3)$
$= 5V^2-(3-2V^2)$
$= \boxed{7V^2-3}$

Question 3. *(6 marks)*
Perform the indicated multiplications.

(a) $-4c^2(-9gc-2c+g^2)$

$= \boxed{36c^3g+8c^3-4c^2g^2}$

(b) $ax(x+4)(7-x^2)$

$= (ax^2+4ax)(7-x^2)$
$= \boxed{7ax^2-ax^4+28ax-4ax^3}$

Question 4. *(6 marks)*
Perform the indicated divisions.

(a) $\frac{9(aB)^4-6aB^4}{3aB^3}$ $= \frac{9a^4B^4}{3aB^3} - \frac{6aB^4}{3aB^3}$

$= \boxed{3a^3B - 2B}$

(b) $\frac{2x^{n+2}+4ax^n}{2x^n}$ $= \frac{2x^{n+2}}{2x^n} + \frac{4ax^n}{2x^n}$

$= \boxed{x^2 + 2a}$

Question 5. *(9 marks)*
Solve the given equations
(a) $3-6(2-3t)=t-5$

$3-12+18t = t-5$

$17t = 4$

$\boxed{t = 4/17}$

(b) $\frac{4x-2(x-4)}{3}=8$

$4x-2x+8 = 24$

$2x = 16$

$\boxed{x = 8}$

(c) $\frac{42}{R}=\frac{7}{3}$ $126 = 7R$

$R = 126/7 = \boxed{18}$

Question 6. *(6 marks)*
Solve the given quadratic equations **by factoring**.

(a) $10b^2 + 23b = 5$

$10b^2 + 23b - 5 = 0$

$10b^2 + 25b - 2b - 5 = 0$

$5b(2b+5) - (2b+5) = 0$

$(2b+5)(5b-1) = 0$

$N \cdot M = -50$

$N + M = 23$

$N = 25$

$M = -2$

so $\boxed{b = -5/2 \ \& \ b = 1/5}$

(b) $x^2 + x - 56 = 0$

$(x+8)(x-7) = 0$

so $\boxed{x = -8 \ \& \ x = 7}$

Question 7. *(6 marks)*
Solve the given quadratic equations by any appropriate algebraic method.
(a) $3x^2 + 8x + 2 = 0$

$$x = \frac{-8 \pm \sqrt{64 - 4(3)(2)}}{2(3)} = \frac{-8 \pm \sqrt{40}}{6}$$

$$= \frac{-8 \pm 2\sqrt{10}}{6}$$

$$= \boxed{-4/3 \pm 1/3\sqrt{10}}$$

(b) $4v^2 = v + 5$

$4v^2 - v - 5 = 0$

$4v^2 - 5v + 4v - 5 = 0$

$v(4v-5) + 1(4v-5) = 0$

$(4v-5)(v+1) = 0$

$\boxed{v = 5/4 \ \& \ v = -1}$

Question 8. *(6 marks)*
A car's radiator contains 12L of antifreeze at a 25% concentration. How many litres must be drained and then replaced by pure antifreeze to bring the concentration to 50% (the manufacturer's "safe" level).

LET x = AMOUNT OF PURE ANTIFREEZE USED (SAME AS AMOUNT OF 25% removed)

y = Amount OF 25% ANTIFREEZE remaining

(A) $x + y = 12$

(B) ANTIFREEZE EQUATION: $x + 0.25y = 0.5(12)$

$x + 0.25(12 - x) = 6$

$x + 3 - 0.25x = 6$

$0.75x = 3$

$x = 4L$

4 LITRES MUST be drained & replaced.

Question 9. Simplify the given expressions. *(5 marks)*

(a) $\sqrt[3]{-27} = \boxed{-3}$ (because $(-3)^3 = -27$)

(b) $16^{-0.5} = \frac{1}{16^{0.5}} = \boxed{\frac{1}{4}}$

(c) $-16^{\frac{3}{2}} = -(\sqrt{16})^3 = -4^3 = \boxed{-64}$

(d) $\sqrt[4]{81} = \boxed{3}$ (b/c $3^4 = 81$)

(e) $\sqrt[3]{-64} = \boxed{-4}$ (because $(-4)^3 = -64$)