

NAME: SOLUTIONS

TEST 3

Dawson College

Applied Math (201-943-DW)

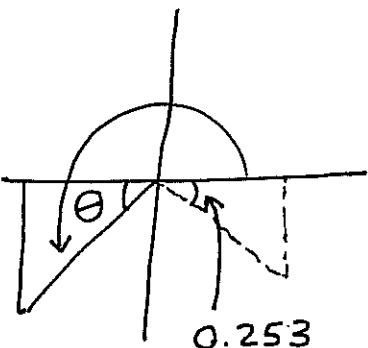
Date: Wednesday December 5th 2012

Instructor: Emilie Richer

This test is marked out of 50 marks

Question 1. (3 marks)

Solve for θ in the equation $\sin \theta = -0.25$ subject to the condition $\tan \theta > 0$. Give all solutions in the range $0 \leq \theta < 2\pi$.



$$\sin^{-1}(-0.25) = -0.253 \text{ rads}$$

$$\begin{aligned}\theta &= \pi + 0.253 \\ &= \boxed{3.39 \text{ rads}}\end{aligned}$$

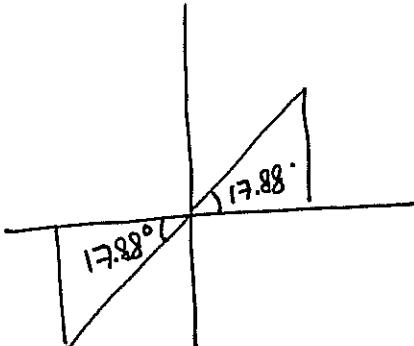
Question 2. (3 marks)

(a) Solve for θ in the equation $\cot \theta = 3.1$. Give all solutions in the range $0^\circ \leq \theta < 360^\circ$.

$$\cot \theta = 3.1$$

$$\tan \theta = \frac{1}{3.1}$$

$$\tan^{-1}\left(\frac{1}{3.1}\right) = 17.88^\circ$$



$$\begin{aligned}\theta_1 &= 17.88^\circ \\ \theta_2 &= 180^\circ + 17.88^\circ \\ &= 197.88^\circ\end{aligned}$$

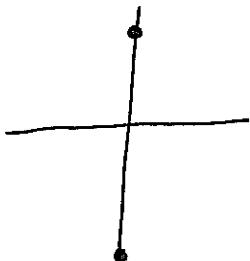
Question 3. (6 marks)

For each equation below, find all values of θ within $0^\circ \leq \theta \leq 360^\circ$.

(a) $2\cos^2 \theta + \cos \theta = 0$

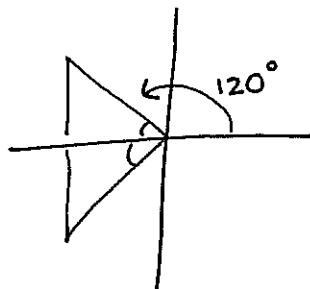
$$\cos \theta (2\cos \theta + 1) = 0$$

$$\cos \theta = 0 \quad \cos \theta = -\frac{1}{2}$$



$$\theta = 90^\circ$$

$$\theta = 270^\circ$$



$$\begin{aligned}\theta &= \cos^{-1}(-\frac{1}{2}) \\ &= 120^\circ\end{aligned}$$

$$\begin{aligned}\theta_2 &= 180^\circ + 60^\circ \\ &= 240^\circ\end{aligned}$$

SOLUTIONS $90^\circ, 270^\circ, 120^\circ, 240^\circ$

(b) $\tan \theta = -\cot \theta + 2$

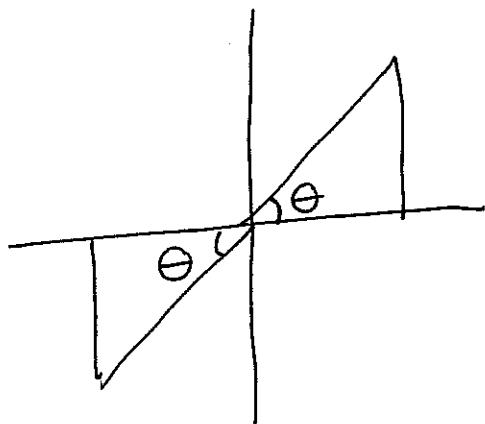
$$\tan \theta = \frac{-1}{\tan \theta} + 2$$

$$\tan^2 \theta = -1 + 2\tan \theta$$

$$\tan^2 \theta - 2\tan \theta + 1 = 0$$

$$(\tan \theta - 1)^2 = 0$$

$$\tan \theta = 1$$



$$\tan^{-1}(1) = 45^\circ$$

$$\theta_1 = 45^\circ$$

$$\theta_2 = 225^\circ$$

Question 4 (5 marks)

Find the values of a and b for which the equation is valid:

$$(a+jb)(4j+5j^2) = j^5(a-1)$$

$$(a+jb)(4j-5) = j(a-1)$$

$$4aj - 5a + 4bj^2 - 5bj = j(a-1)$$

$$\underbrace{-5a-4b}_{\text{real}} + \underbrace{(4a-5b)j}_{\text{IMAGINARY}} = (a-1)j$$

EQUATING REAL & IMAGINARY PARTS
WE GET:

$$-5a-4b=0 \quad ①$$

$$4a-5b=a-1$$

$$3a-5b=-1 \quad ②$$

$$\text{ISOLATE } a \text{ in } ① \quad a = \frac{4b}{-5}$$

Sub in ②

$$3\left(\frac{4b}{-5}\right) - 5b = -1$$

$$\frac{-12}{5}b - 5b = -1$$

$$-\frac{37}{5}b = -1$$

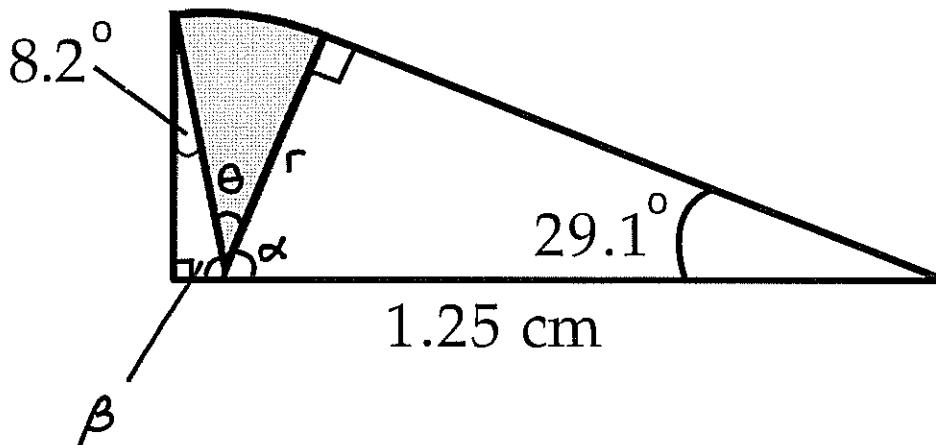
$$\rightarrow b = \frac{5}{37}$$

$$\rightarrow a = \frac{4}{-5} \left(\frac{5}{37} \right) = -\frac{4}{37}$$

$$[a = -\frac{4}{37}]$$

Question 5. (5 marks)

Find the area of the sector of a circle shaded in the diagram below.



- we find β : $180^\circ - 90^\circ - 8.2^\circ = 81.8^\circ$
- we find α : $- 29.1^\circ = 60.9^\circ$
- we can now find θ :
$$\theta = 180^\circ - 81.8^\circ - 60.9^\circ$$

$$= 37.3^\circ$$

IN radians: 0.651

- we find r : $\sin 29.1^\circ = \frac{r}{1.25}$

$$r = 0.6079 \text{ cm}$$
- now we can find Area:

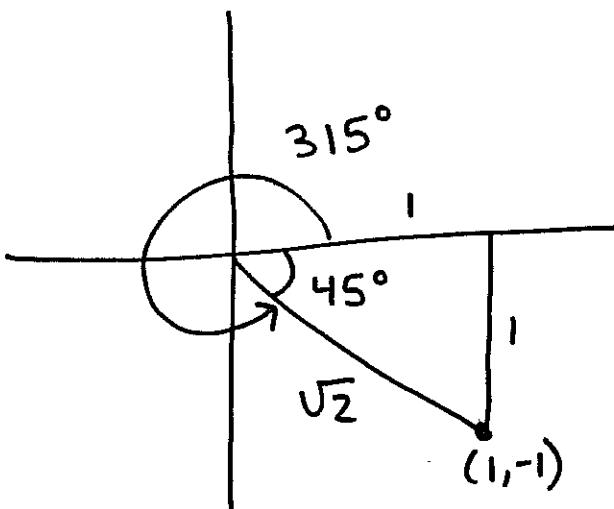
$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (0.6079)^2 (0.651)$$

$$= \boxed{0.120 \text{ cm}^2}$$

Question 6. (4 marks)

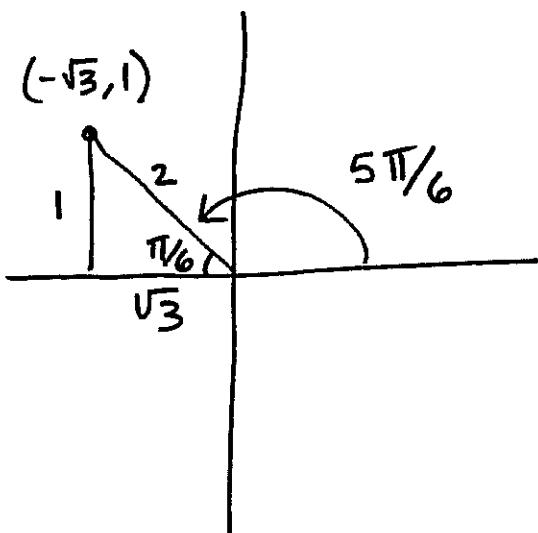
Using "special triangles" find the **EXACT VALUE** of the following. Show all your work including the positioning of the special triangles in the appropriate quadrants.

(a) $\cos(315^\circ)$



$$\begin{aligned}\cos 315^\circ &= x/h \\ &= 1/\sqrt{2}\end{aligned}$$

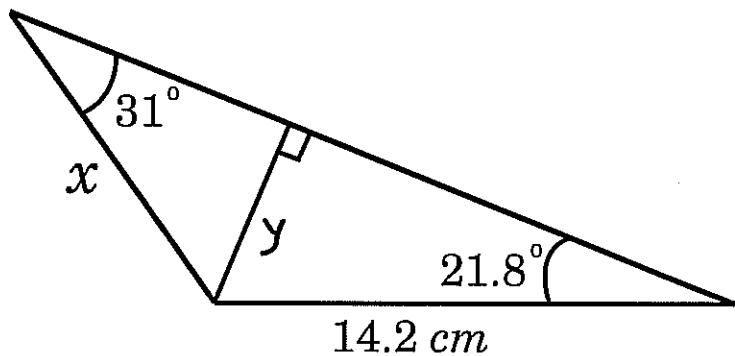
(b) $\tan(\frac{5\pi}{6})$



$$\tan \frac{5\pi}{6} = \frac{1}{-\sqrt{3}}$$

Question 7. (4 marks)

Find the length x in the diagram given below.



• we find y :

$$\sin 21.8^\circ = \frac{y}{14.2}$$

$$y = 5.27 \text{ cm}$$

• now x :

$$\sin 31^\circ = \frac{y}{x}$$

$$x = \frac{5.27}{\sin 31^\circ}$$

$$= \boxed{10.23 \text{ cm}}$$

Question 8. (2.5 marks each = 10 marks)

Perform the indicated operations, expressing all answers in simplest rectangular form.

$$(a) (-1 + \sqrt{-9}) - (-2 - 6j)$$

$$= (-1 + 3j) + 2 + 6j$$

$$= \boxed{1 + 9j}$$

$$(b) (1 - 2j) - 2j^3 \sqrt{-36}$$

$$= (1 - 2j) + 2j(6j)$$

$$= 1 - 2j + 12j^2 = 1 - 2j - 12$$

$$= \boxed{-11 - 2j}$$

$$(c) j^2(-1 + 2j) - (j^7)(-5 + j)$$

$$= -j^2 + 2j^3 - (-j)(-5 + j)$$

$$= 1 - 2j - 5j + j^2$$

$$= \boxed{-7j - 1}$$

$$= \boxed{-7j}$$

$$(d) \frac{-1+6j}{1-\sqrt{-4}}$$

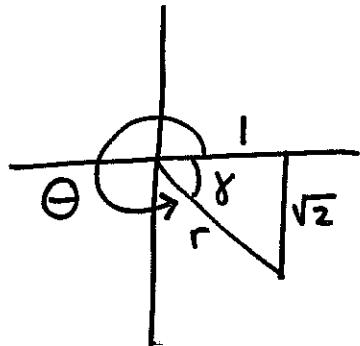
$$= (-1 + 6j) \left(\frac{1+2j}{1-2j} \right)$$

$$= \frac{-1 + 6j - 2j + 12j^2}{1 - 4j^2}$$

$$= \boxed{\frac{-13 + 4j}{5}}$$

Question 9. (3 marks each = 6 marks)

(a) Give the polar and exponential form of the complex number $1 - \sqrt{2}j$



$$r^2 = (\sqrt{2})^2 + 1^2$$

$$= 3$$

$$r = \sqrt{3}$$

$$\gamma = \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = 54.7^\circ$$

$$\Theta = 360^\circ - 54.7^\circ = 305.26^\circ, 5.328 \text{ rad}$$

Polar Form: $\sqrt{3}/\underline{305.26^\circ}$

Exp. Form: $\sqrt{3} e^{5.328j}$

(b) Give the exponential and rectangular form of the complex number $5/\underline{180^\circ}$

Rectangular: $5 \cos 180^\circ + 5 \sin 180^\circ j$
= -5

Exp: $5 e^{\pi j}$

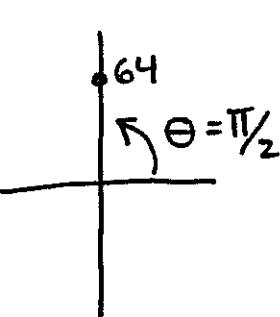
Question 10. (4 marks)

Find all the three solutions (real and complex) of the given equation. Express the final result in rectangular and polar forms.

$$x^3 - 64j = 0$$

$$X^3 = 64j$$

64j in exp form:



$$\begin{array}{l} 64 \\ \nwarrow \theta = \frac{\pi}{2} \end{array}$$

① $64e^{\frac{\pi i}{2}}$ ② $64e^{\frac{5\pi i}{2}}$ ③ $64e^{\frac{9\pi i}{2}}$

APPLY EXPONENT $\frac{1}{3}$

$$\begin{array}{l} ① 4e^{\frac{\pi i}{6}} \quad ② 4e^{\frac{5\pi i}{6}} \quad ③ 4e^{\frac{9\pi i}{6}} \end{array}$$

POLAR FORMS

$$\boxed{① 4 \angle 30^\circ}$$

$$\boxed{② 4 \angle 150^\circ}$$

$$\boxed{③ 4 \angle 270^\circ}$$

RECTANGULAR FORMS

$$\begin{array}{l} ① 4\cos 30^\circ + 4\sin 30^\circ j \quad ② \end{array}$$

$$= 4\left(\frac{\sqrt{3}}{2}\right)j + 4\left(\frac{1}{2}\right)j$$

$$= \boxed{2\sqrt{3}j + 2j}$$

$$= 4\left(-\frac{\sqrt{3}}{2}\right)j + 4\left(\frac{1}{2}\right)j \quad ③$$

$$= \boxed{-2\sqrt{3}j + 2j}$$

$$= \boxed{-4j}$$