Dawson College: **Calculus III**: 201-BZF-05 S01 October 2, 2012

Last Name: SOLUTIONS

First Name: ____________

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Quiz 3

Question 1. *(5 marks)* Use the definition of the Maclaurin series to find the Maclaurin series for $f(x) = 2^x$. Find the radius of convergence of this series.

$$f(x) = 2^x \qquad f(0) = 1$$

$$f'(x) = 2^x \ln 2 \quad \Rightarrow \quad f'(0) = \ln 2$$

$$f''(x) = 2^x (\ln 2)^2 \qquad f''(0) = (\ln 2)^2$$

$$\vdots \qquad\qquad \vdots$$

$$f^{(n)}(x) = 2^x (\ln 2)^n \qquad f^{(n)}(0) = (\ln 2)^n$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} x^n \leftarrow \text{MACLAURIN SERIES}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(\ln 2)^{n+1}}{(n+1)!} \cdot \frac{n!}{(\ln 2)^n} \right| = \lim_{n\to\infty} \frac{\ln 2}{n+1} = 0 < 1$$

$\therefore$ THE RADIUS OF CONVERGENCE IS $R = \infty$. (BY RATIO TEST)

Question 2. *(5 marks)* Find the limit of the vector function

$$\lim_{t\to\infty}\left\langle \frac{1+t^2}{1-t^2}, \arctan t, \frac{1-e^{-2t}}{t}\right\rangle$$

$$= \left\langle \lim_{t\to\infty} \frac{1+t^2}{1-t^2}, \lim_{t\to\infty} \arctan t, \lim_{t\to\infty} \frac{1-e^{-2t}}{t} \right\rangle$$

$$= \left\langle -1, \pi/2, \lim_{t\to\infty} \frac{e^{2t}-1}{e^{2t}\cdot t} \right\rangle$$

$$= \left\langle -1, \pi/2, \lim_{t\to\infty} \left(\frac{1}{t} - \frac{1}{e^{2t}\cdot t}\right) \right\rangle$$

$$= \langle -1, \pi/2, 0 \rangle$$