October 9, 2012

Last Name: SOLUTIONS

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Quiz 4

Question 1. *(5 marks)* Find the parametric equations for the tangent line to the curve $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$ at the point $(1,0,1)$. $\Leftrightarrow t = 0$

$\vec{r}'(t) = \langle -e^{-t}\cos t - e^{-t}\sin t,\ -e^{-t}\sin t + e^{-t}\cos t,\ -e^{-t} \rangle$

$\vec{r}'(0) = \langle -1, 1, -1 \rangle$

$\therefore$ TANGENT LINE: $x = 1 - t$, $y = t$, $z = 1 - t$

Question 2. *(5 marks)* Find the length of the curve given by $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ when $0 \le t \le 1$.

$\vec{r}\,'(t) = \langle 0, 2t, 3t^2 \rangle$

$|\vec{r}\,'(t)| = \sqrt{4t^2 + 9t^4} = \sqrt{t^2}\sqrt{4 + 9t^2}$

$= |t|\sqrt{4+9t^2} = t\sqrt{4+9t^2}$ SINCE $0 \le t \le 1$

$\therefore L = \int_0^1 |\vec{r}\,'(t)|\,dt = \int_0^1 t\sqrt{4+9t^2}\,dt$

LET $u = 4 + 9t^2$

$du = 18t\,dt$

IF $t = 0 \Rightarrow u = 4$

$t = 1 \Rightarrow u = 13$

$= \int_4^{13} t\sqrt{u}\,\frac{du}{18t} = \frac{1}{18}\int_4^{13} u^{1/2}\,du$

$= \frac{1}{18} \cdot \frac{u^{3/2}}{3/2}\Big|_4^{13} = \frac{1}{27}u^{3/2}\Big|_4^{13} = \frac{1}{27}\left[13^{3/2} - 4^{3/2}\right]$

$= \frac{1}{27}\left[13^{3/2} - 8\right]$