Dawson College: Calculus III: 201-BZF-05 S01 October 16, 2012

Last Name: SOLUTIONS

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Quiz 5

Question 1. *(10 marks)* Find **T**, **N**, **B**, the normal plane and the osculating plane to $x = 2\sin(3t)$, $y = t$, $z = 2\cos(3t)$ at the point $(0, \pi, -2)$.

$\vec{r}\,'(t) = \langle 6\cos(3t), 1, -6\sin(3t) \rangle$, $|\vec{r}\,'(t)| = \sqrt{36\cos^2(3t) + 1 + 36\sin^2(3t)}$

$= \sqrt{36+1}$

$= \sqrt{37}$

$\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \dfrac{1}{\sqrt{37}} \langle 6\cos(3t), 1, -6\sin(3t) \rangle$

$\vec{T}\,'(t) = \dfrac{1}{\sqrt{37}} \langle -18\sin(3t), 0, -18\cos(3t) \rangle$

$|\vec{T}\,'(t)| = \sqrt{\left(\dfrac{1}{\sqrt{37}}\right)^2 (18)^2 \sin^2(3t) + \left(\dfrac{1}{\sqrt{37}}\right)^2 (18)^2 \cos^2(3t)}$

$= \dfrac{1}{\sqrt{37}}(18)$

$\therefore \vec{N}(t) = \dfrac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|} = \dfrac{1}{18} \langle -18\sin(3t), 0, -18\cos(3t) \rangle$

$= \langle -\sin(3t), 0, -\cos(3t) \rangle$

SO, AT $(0, \pi, -2) \Rightarrow t = \pi$

$\vec{T}(\pi) = \dfrac{1}{\sqrt{37}} \langle -6, 1, 0 \rangle$, $\vec{N}(\pi) = \langle 0, 0, 1 \rangle$

$\therefore \vec{B}(\pi) = \vec{T}(\pi) \times \vec{N}(\pi) = \dfrac{1}{\sqrt{37}} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \dfrac{1}{\sqrt{37}} \left(\vec{i} - (-6)\vec{j} + 0\vec{k} \right)$

$= \dfrac{1}{\sqrt{37}} \langle 1, 6, 0 \rangle$

NORMAL PLANE: $\vec{n} = \langle -6, 1, 0 \rangle \Rightarrow -6x + y + d = 0$

$\Rightarrow -6(0) + \pi + d = 0 \Rightarrow d = -\pi$

$\therefore \boxed{-6x + y - \pi = 0}$

OSCULATING PLANE: $\vec{n} = \langle 1, 6, 0 \rangle \Rightarrow x + 6y + d = 0$

$0 + 6\pi + d = 0 \Rightarrow d = -6\pi$

$\therefore \boxed{x + 6y - 6\pi = 0}$