

Last Name: SOLUTIONS

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Quiz 7

Question 1. (5 marks) Find f_{xyz} for $f(x,y,z) = e^{xyz^2}$

$$f_x = yz^2 e^{xyz^2}$$

$$f_{xy} = (f_x)_y = z^2 e^{xyz^2} + yz^2 \cdot xz^2 e^{xyz^2}$$

$$= z^2 e^{xyz^2} + xyz^4 e^{xyz^2}$$

$$f_{xyz} = (f_{xy})_z = 2ze^{xyz^2} + 4xyz^3 e^{xyz^2} + xyz^4 \cdot xy \cdot 2ze^{xyz^2}$$

$$+ z^2 e^{xyz^2} \cdot 2xyz$$

$$= 2ze^{xyz^2} + 6xyz^3 e^{xyz^2} + 2x^2 y^2 z^5 e^{xyz^2}$$

Question 2. (5 marks) Find the linearization of $f(x,y) = 1 + x \ln(xy-5)$ at $(2,3)$.

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$L(x,y) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + z_0$$

$$f_x(x,y) = \ln(xy-5) + x \cdot \frac{1}{xy-5} \cdot y$$

$$= \ln(xy-5) + \frac{xy}{xy-5}$$

$$\therefore f_x(2,3) = \ln(6-5) + \frac{6}{6-5} = \ln(1) + 6 = 6$$

$$f_y(x,y) = \frac{x}{xy-5} \cdot x = \frac{x^2}{xy-5}$$

$$f_y(2,3) = \frac{(2)^2}{6-5} = 4$$

$$z_0 = f(2,3) = 1 + 2 \ln(6-5) = 1$$

$$\begin{aligned} \therefore L(x,y) &= 6(x-2) + 4(y-3) + 1 \\ &= 6x - 12 + 4y - 12 + 1 \\ &= 6x + 4y - 23 \end{aligned}$$