Dawson College: Calculus III: 201-BZF-05 S01

November 13, 2012

Last Name: SOLUTIONS

First Name:

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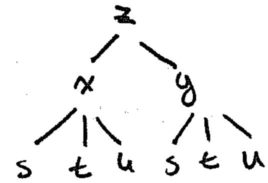
Quiz 8

Question 1. *(5 marks)* Given $z = x^4 + x^2y$ and $x = s + 2t - u$, $y = stu^2$ find $\frac{\partial z}{\partial u}$ when $s = 4$, $t = 2$, $u = 1$.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$



$$= (4x^3 + 2xy)(-1) + (x^2)(2stu)$$

WHEN $s=4, t=2, u=1 \Rightarrow x = 4 + 2(2) - 1 = 7$, $y = 4(2)(1)^2 = 8$

SO $\frac{\partial z}{\partial u} = \left[4(7)^3 + 2(7)(8)\right](-1) + (7)^2(2(4)(2)(1))$

$$= -700$$

Question 2. *(5 marks)* Find the gradient of $f(x,y,z) = x^2yz - xyz^3$ at P$(2,-1,1)$. Use it to find the rate of change of f at P in the direction of $\vec{\mathbf{u}} = \langle 0, \frac{4}{5}, -\frac{3}{5} \rangle$

$f_x = 2xyz - yz^3 \Rightarrow f_x(2,-1,1) = 2(2)(-1)(1) - (-1)(1)^3 = -3$

$f_y = x^2z - xz^3 \Rightarrow f_y(2,-1,1) = (2)^2(1) - (2)(1)^3 = 2$

$f_z = x^2y - 3xyz^2 \Rightarrow f_z(2,-1,1) = (2)^2(-1) - 3(2)(-1)(1)^2 = 2$

$|\vec{u}| = \sqrt{0^2 + \left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = 1 \quad \therefore$ $\vec{u}$ IS A UNIT VECTOR.

$\nabla f(2,-1,1) = \langle -3, 2, 2 \rangle$

$D_{\vec{u}} f(2,-1,1) = \langle -3, 2, 2 \rangle \cdot \langle 0, \frac{4}{5}, -\frac{3}{5} \rangle$

$= \frac{8}{5} - \frac{6}{5} = \frac{2}{5}$