

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 9

Question 1. (10 marks) Find the local maximum and minimum values and the saddle points of the function $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$

$$f_x = 6xy - 12x, \quad f_y = 3y^2 + 3x^2 - 12y$$

$$f_x = 0 \Rightarrow 6x(y-2) = 0 \Rightarrow x=0 \quad \text{or} \quad y=2$$

If $x=0$ $\Rightarrow 0 = f_y = 3y^2 + 3(0)^2 - 12y = 3y(y-4) \Rightarrow y=0, 4$

$\therefore (0,0)$ AND $(0,4)$ ARE C.P.

If $y=2$ $\Rightarrow 0 = f_x = 12 + 3x^2 - 24 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

SO $(-2, 2)$ AND $(2, 2)$ ARE C.P.

$$f_{xx} = 6y - 12, \quad f_{yy} = 6y - 12, \quad f_{xy} = 6x$$

$$\text{SO } D(x,y) = (6y-12)(6y-12) - [6x]^2 = (6y-12)^2 - (6x)^2$$

SO

(x,y)	f	f_{xx}	D	CONCLUSION
$(0,0)$	2	$-12 < 0$	$144 > 0$	LOCAL MAXIMUM
$(0,4)$	-30	$12 > 0$	$144 > 0$	LOCAL MINIMUM
$(-2,2)$	-14	/	$-144 < 0$	SADDLE POINT
$(2,2)$	-14	/	$-144 < 0$	SADDLE POINT