Last Name: ____________________

First Name: ____________________

Student ID: ____________________

Test 2

The length of the test is 1hr and 45min. The test consistst of 9 questions. Complete any 8 of the 9 questions during the test. **Hand in the 9th question of your choice at the beginning of tomorrow's class**. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**. Do not use decimals unless otherwise stated.

Question 1. *(5 marks)* Use the definition of a Maclaurin series to find the Maclaurin series for the function $f(x) = \ln(1+3x)$.

Question 2. *(5 marks)* Find the limit of the vector function $\vec{r}(t)$ below.

$$\lim_{t\to\infty}\left\langle 2\arctan t,\ (t+2)e^{-t},\ \frac{3t^2-1}{2t^2+4}\right\rangle$$

What does this limit mean geometrically?

Question 3. *(5 marks)* Find the vector function that represents the curve of intersection of the sphere $x^2+y^2+z^2=25$ and the plane $z=4$. Graph this curve.

Question 4. Given $\vec{\mathbf{r}}(t) = \langle \cos 8t, t, \sin 8t \rangle$

(a) *(5 marks)* Find $\vec{\mathbf{T}}$, $\vec{\mathbf{N}}$, $\vec{\mathbf{B}}$ to $\vec{\mathbf{r}}(t)$ at $(1, \pi, 0)$.

(a) *(3 marks)* Find the normal plane and the osculating plane to $\vec{\mathbf{r}}(t)$ at $(1, \pi, 0)$.

Question 5. *(5 marks)* Find the curvature of $f(x) = 4 + 2x - 3x^2$ and use it to find the point on the curve with maximum curvature.

Question 6. *(6 marks)* Find the arc length function for

$$\vec{\mathbf{r}}(t) = \left\langle \frac{1}{2}t^2,\ 3t,\ \frac{2\sqrt{6}}{3}t^{3/2} \right\rangle$$

measured from $t = 0$ in the direction of increasing t (assume $t \geq 0$). Find the point (a,b,c) on this curve where the length from $(0,0,0)$ to (a,b,c) is 4. (Note: you do not have to reprametrize the curve with respect to arc length to solve this problem.)

Question 7. *(6 marks)* A moving particle has velocity function

$$\vec{v}(t) = \left\langle \sqrt{1+3t},\ \frac{1}{t+1},\ te^{t} \right\rangle$$

Find the position function $\vec{s}(t)$ and the acceleration function $\vec{a}(t)$ for the particle given that $\vec{s}(0) = (\frac{2}{9}, 1, 2)$.

Question 8. *(4 marks)* Sketch the graph of the plane $z = 3x + 2y + 4z - 12$ in the first octant ($x, y, z \geq 0$).

Question 9: *(6 marks)* Determine the set of points where the function

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous. Explain your reasoning.

$$\kappa(t) = \frac{|\vec{\mathbf{T}}'(t)|}{|\vec{\mathbf{r}}'(t)|}$$

$$\kappa(t) = \frac{|\vec{\mathbf{r}}'(t) \times \vec{\mathbf{r}}''(t)|}{|\vec{\mathbf{r}}'(t)|^3}$$

$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$$

$$a_T = \frac{\vec{\mathbf{r}}'(t) \cdot \vec{\mathbf{r}}''(t)}{|\vec{\mathbf{r}}'(t)|}$$

$$a_N = \frac{|\vec{\mathbf{r}}'(t) \times \vec{\mathbf{r}}''(t)|}{|\vec{\mathbf{r}}'(t)|}$$