Dawson College: **Calculus II**: 201-203-DW S03

September 14, 2012

Last Name: SOLUTIONS

First Name: __________

Student ID: __________

Quiz 1

Question 1. Find the indefinite integral:

(a) *(4 marks)* $\int \frac{(\sqrt{x}-1)^2}{x^2}\,dx = \int \frac{x-\sqrt{x}-\sqrt{x}+1}{x^2}\,dx$

$$= \int \frac{x-2\sqrt{x}+1}{x^2}\,dx = \int \frac{x}{x^2} - \frac{2x^{1/2}}{x^2} + \frac{1}{x^2}\,dx$$

$$= \int x^{-1} - 2x^{-3/2} + x^{-2}\,dx = \ln|x| - 2\frac{x^{-1/2}}{-1/2} + \frac{x^{-1}}{-1} + c$$

$$= \ln|x| + \frac{4}{x^{1/2}} - \frac{1}{x} + c$$

(b) *(5 marks)* $\int \frac{e^{3x}+x^2}{(e^{3x}+x^3)^3}\,dx$

LET $u = e^{3x}+x^3$

$du = (3e^{3x}+3x^2)\,dx$

$\frac{du}{3e^{3x}+3x^2} = dx$

$$= \int \frac{e^{3x}+x^2}{u^3}\,\frac{du}{3e^{3x}+3x^2}$$

$$= \int \frac{e^{3x}+x^2}{u^3}\cdot\frac{du}{3(e^{3x}+x^2)} = \frac{1}{3}\int u^{-3}\,du$$

$$= \frac{1}{3}\,\frac{u^{-2}}{-2} + C = -\frac{1}{6}u^{-2} + c = -\frac{1}{6}\cdot\frac{1}{(e^{3x}+x^3)^2} + c$$

(c) *(5 marks)* $\int x^2\sqrt{2x-1}\,dx$

LET $u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$u+1 = 2x$

$\frac{u+1}{2} = x$

$= \int \left(\frac{u+1}{2}\right)^2 \sqrt{u}\,\frac{du}{2} = \int \frac{u^2+2u+1}{4} \cdot u^{1/2}\,\frac{du}{2}$

$= \frac{1}{8}\int (u^2+2u+1)u^{1/2}\,du = \frac{1}{8}\int u^{5/2} + 2u^{3/2} + u^{1/2}\,du$

$= \frac{1}{8}\left[\frac{u^{7/2}}{7/2} + \frac{2u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right] + C \quad = \frac{1}{28}u^{7/2} + \frac{1}{10}u^{5/2} + \frac{1}{12}u^{3/2} + c$

$= \frac{1}{28}(2x-1)^{7/2} + \frac{1}{10}(2x-1)^{5/2} + \frac{1}{12}(2x-1)^{3/2} + c$

Question 2 *(4 marks)* Find $f(x)$ given $f(1) = 2$ and

$f'(x) = 1 + \frac{1}{x^2}$

$f(x) = \int f'(x)\,dx = \int 1 + x^{-2}\,dx = x + \frac{x^{-1}}{-1} + c = x - \frac{1}{x} + c$

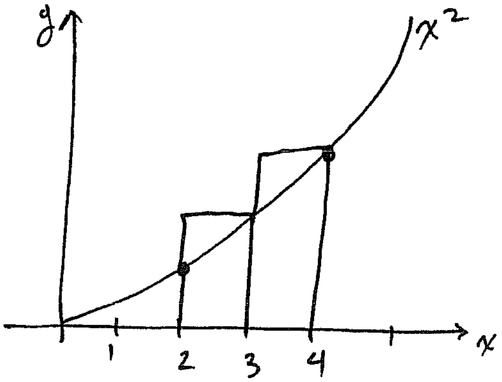
$2 = f(1) = 1 - \frac{1}{1} + c$

$2 = c$

$\therefore \quad f(x) = x - \frac{1}{x} + 2$

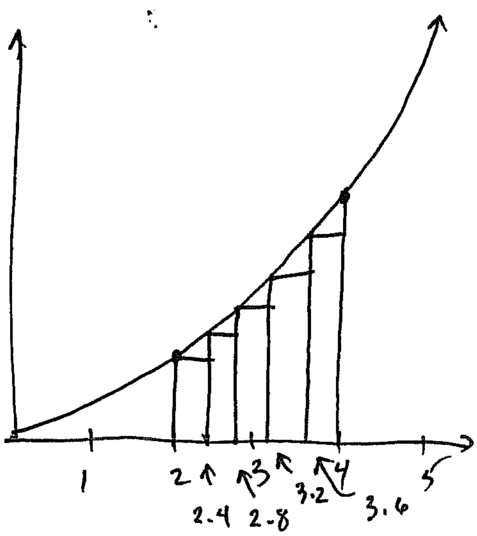
Question 3. *(6 marks)* Let $f(x) = x^2$ and compute the Riemann sum (sum of approximating rectangles) of f over the interval $[2,4]$ using

(a) Two subintervals (n=2) and choose the representative points to be the right endpoints.



$$R_2 = 1 \cdot f(3) + 1 \cdot f(4) = 3^2 + 4^2 = 25$$

(a) Five subintervals (n=5) and choose the representative points to be the left endpoints.



$$\Delta x = \frac{4-2}{5} = \frac{2}{5}$$

$$\therefore L_5 = \frac{2}{5} f(2) + \frac{2}{5} f(2.4) + \frac{2}{5} f(2.8) + \frac{2}{5} f(3.2) + \frac{2}{5} f(3.6)$$

$$= \frac{2}{5}(2)^2 + \frac{2}{5}(2.4)^2 + \frac{2}{5}(2.8)^2 + \frac{2}{5}(3.2)^2 + \frac{2}{5}(3.6)^2$$

$$= 16.32$$

Question 4. *(6 marks)* Use the definition of the definite integral to evaluate

$$\int_1^5 (1-4x^2)\,dx$$

• $\Delta x = \dfrac{b-a}{n} = \dfrac{5-1}{n} = \dfrac{4}{n}$ • $x_i = a + i\Delta x = 1 + \dfrac{4i}{n}$

• $f(x_i) = 1 + 4x_i^2 = 1 - 4\left(1 + \dfrac{4i}{n}\right)^2 = 1 - 4\left(1 + \dfrac{8i}{n} + \dfrac{16i^2}{n^2}\right)$

$= 1 - 4 - \dfrac{32i}{n} - \dfrac{64i^2}{n^2} = -3 - \dfrac{32i}{n} - \dfrac{64i^2}{n^2}$

• $f(x_i)\Delta x = \left(-3 - \dfrac{32i}{n} - \dfrac{64i^2}{n^2}\right)\left(\dfrac{4}{n}\right) = \dfrac{-12}{n} - \dfrac{128i}{n^2} - \dfrac{256i^2}{n^3}$

• $\displaystyle\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(\frac{-12}{n} - \frac{128i}{n^2} - \frac{256i^2}{n^3}\right) = -\frac{12}{n}\sum_{i=1}^{n} 1 - \frac{128}{n^2}\sum_{i=1}^{n} i - \frac{256}{n^3}\sum_{i=1}^{n} i^2$

$= -\dfrac{12}{n}\cdot n - \dfrac{128}{n^2}\cdot\dfrac{n(n+1)}{2} - \dfrac{256}{n^3}\cdot\dfrac{n(n+1)(2n+1)}{6}$

$= -12 - 64\cdot\dfrac{(n+1)}{n} - \dfrac{128}{3}\cdot\dfrac{2n^2+3n+1}{n^2}$

• $\displaystyle\int_1^5 (1-4x^2)\,dx = \lim_{n\to\infty}\left[-12 - 64\left(\frac{1+1/n}{1}\right) - \frac{128}{3}\cdot\frac{2+3/n+1/n^2}{1}\right]$

$= -12 - 64\left(\dfrac{1+0}{1}\right) - \dfrac{128}{3}\cdot\dfrac{2+0+0}{1}$

$= -12 - 64 - \dfrac{256}{3}$

$= \dfrac{-484}{3}$