Dawson College: Calculus II: 201-203-DW S03

October 19, 2012

Last Name: SOLUTIONS

First Name: ______________

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Quiz 2

Question 1. Find the following integrals:

(a) *(4 marks)* $\int \frac{\ln x}{x^3}\,dx$

LET $u = \ln x$ $\quad dv = \frac{1}{x^3}dx$

$du = \frac{1}{x}dx \quad v = -\frac{1}{2x^2}$

$$uv - \int v\,du = -\frac{1}{2x^2}\ln x - \int\left(-\frac{1}{2x^2}\right)\cdot\frac{1}{x}\,dx$$

$$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\int x^{-3}dx = -\frac{1}{2x^2}\ln x + \frac{1}{2}\cdot\frac{x^{-2}}{-2} + C$$

$$= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + C$$

(b) *(5 marks)* $\int \sqrt{x}\,\ln\sqrt{x}\,dx$

LET $u = \ln\sqrt{x}$ $\quad dv = \sqrt{x}\,dx$

$du = \frac{1}{\sqrt{x}}\cdot\frac{x^{-1/2}}{2}dx \quad v = \frac{2}{3}x^{3/2}$

$= \frac{1}{2x}dx$

$$uv - \int v\,du = \frac{2}{3}x^{3/2}\ln\sqrt{x} - \int \frac{2}{3}x^{3/2}\cdot\frac{1}{2x}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln\sqrt{x} - \frac{1}{3}\int x^{1/2}dx$$

$$= \frac{2}{3}x^{3/2}\ln\sqrt{x} - \frac{1}{3}\cdot\frac{x^{3/2}}{3/2} + C$$

$$= \frac{2}{3}x^{3/2}\ln\sqrt{x} - \frac{2}{9}x^{3/2} + C$$

(c) *(5 marks)* $\int_0^2 xe^{2x}\,dx$

LET $u = x$ $\quad dv = e^{2x}dx$

$du = dx$ $\quad v = \underbrace{\frac{1}{2}e^{2x}}_{\text{SUBSTITUTION}}$

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$$= \frac{1}{2}xe^{2x}\Big|_0^2 - \int_0^2 \frac{1}{2}e^{2x}dx$$

$$= \frac{1}{2}xe^{2x}\Big|_0^2 - \frac{1}{4}e^{2x}\Big|_0^2$$

$$= \frac{1}{2}(2)e^4 - \frac{1}{2}(0)e^0 - \left[\frac{1}{4}e^4 - \frac{1}{4}e^0\right]$$

$$= e^4 - \frac{1}{4}e^4 + \frac{1}{4}$$

$$= \frac{3}{4}e^4 + \frac{1}{4}$$

Question 2. *(6 marks)* The quantity demanded x (in units of a hundred) of the Mikado miniature cameras/week is related to the unit price p (in dollars) by

$$p = -0.2x^2 + 80$$

and the quantity x (in units of a hundred) that the supplier is willing to make available in the market is related to the unit price p (in dollars)by

$$p = 0.1x^2 + x + 40$$

If the market price is set at the equilibrium price, find the consumer's surplus and the producer's surplus.

FINDING EQUILIBRIUM:

$$-0.2x^2 + 80 = 0.1x^2 + x + 40$$

$$0 = 0.3x^2 + x - 40$$

$$0 = 3x^2 + 10x - 400$$

$$0 = 3x^2 + 40x - 30x - 400$$

$$0 = x(3x+40) - 10(3x+40)$$

$$0 = (x-10)(3x+40)$$

$$\therefore \bar{x} = 10, \quad \cancel{x = -\frac{40}{3}}$$

$$\text{so } \bar{p} = -0.2(10)^2 + 80$$

$$= 60$$

SO

$$CS = \int_0^{\bar{x}} D(x) - \bar{p}\bar{x}$$

$$= \int_0^{10} (-0.2x^2 + 80)\,dx - (60)(10)$$

$$= \left[\frac{-0.2x^3}{3} + 80x\right]_0^{10} - 600$$

$$= \left[\left(\frac{-0.2(10)^3}{3} + 80(10)\right) - (0)\right] - 600$$

$$= 133.\bar{3}$$

$\therefore$ THE CONSUMERS' SURPLUS IS

\$ 13 333.33

$$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} s(x)\,dx = (60)(10) - \int_0^{10} 0.1x^2 + x + 40\,dx$$

$$= 600 - \left[\frac{0.1x^3}{3} + \frac{x^2}{2} + 40x\right]_0^{10} = 600 - \left[\left(\frac{0.1(10)^3}{3} + \frac{(10)^2}{2} + 40(10)\right) - (0)\right]$$

$$= 116.\bar{6}$$

$\therefore$ THE PRODUCERS' SURPLUS IS \$ 11666.67

Question 3. Find the following integrals:

(a) *(5 marks)* $\displaystyle\int \frac{12x+18}{(x+3)(2x^2+8x+9)}\,dx$ $= I$

$$\frac{12x+18}{(x+3)(2x^2+8x+9)} = \frac{A}{x+3} + \frac{Bx+C}{2x^2+8x+9}$$

$$12x+18 = A(2x^2+8x+9) + (Bx+C)(x+3)$$

IF $\underline{x=-3}$

$-36+18 = A(18-24+9) + 0$

$-18 = 3A$

$\therefore$ $\boxed{A=-6}$

IF $\underline{x=0}$

$18 = -6(9) + (0+C)(0+3)$

$18 = -54 + 3C$

$\boxed{24=C}$

IF $\underline{x=1}$

$12+18 = -6(2+8+9) + (B+24)(1+3)$

$30 = -114 + 4B + 96$

$48 = 4B$ $\therefore$ $\boxed{B=12}$

$$\therefore\ I = \int \frac{-6}{x+3} + \frac{12x+24}{2x^2+8x+9}\,dx$$

LET $u = 2x^2+8x+9$

$du = (4x+8)\,dx$

$\dfrac{du}{4(x+2)} = dx$

$$= -6\ln|x+3| + 12\int \frac{x+2}{u}\cdot\frac{du}{4(x+2)}$$

$$= -6\ln|x+3| + 3\int \frac{1}{u}\,du$$

$$= -6\ln|x+3| + 3\ln|2x^2+8x+9| + C$$

(b) *(4 marks)* $\int \frac{5x^3+3x-2}{x-1}\,dx = I$

$$
\begin{array}{r|l}
 & 5x^2+5x+8 \\
x-1 & 5x^3+0x^2+3x-2 \\
 & -(5x^3-5x^2) \\
 & \quad 5x^2+3x \\
 & \quad -(5x^2-5x) \\
 & \qquad 8x-2 \\
 & \qquad -(8x-8) \\
 & \qquad\quad 6
\end{array}
$$

$$\therefore \quad I = \int 5x^2+5x+8+\frac{6}{x-1}\,dx$$

$$= \frac{5}{3}x^3+\frac{5}{2}x^2+8x+6\ln|x-1|+C$$