Dawson College: **Calculus II - Social Science**: 201-203-DW S03 November 2, 2012

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 2

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**.

Question 1. *(4 marks)* Use the method of partial fractions to write the following as a sum of simpler fractions. You do not have to solve for the variables A, B, C, ... etc.

$$\frac{4x^3-2x+5}{x^2(3x-2)^3(x^2+5x+4)(x^2+2x+1)}$$

$x^2+5x+4 = (x+4)(x+1)$

$(x+1)(x+1)$

$$= \frac{4x^3-2x+5}{x^2(3x-2)^3(x+4)(x+1)(x+1)^2}$$

$$= \frac{4x^3-2x+5}{x^2(3x-2)^3(x+4)(x+1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x-2} + \frac{D}{(3x-2)^2} + \frac{E}{(3x-2)^3} + \frac{F}{x+4} + \frac{G}{x+1} + \frac{H}{(x+1)^2} + \frac{I}{(x+1)^3}$$

Question 2. Find the following integrals:

(a) *(5 marks)* $\int x^3 \ln(x^2)\, dx$

LET $u = \ln(x^2)$ $\quad dv = x^3\, dx$

$du = \frac{1}{x^2} \cdot 2x\, dx$ $\quad v = \frac{1}{4}x^4$

$= \frac{2}{x} dx$

$= uv - \int v\, du$

$= \frac{1}{4}x^4 \ln(x^2) - \int \frac{1}{4}x^4 \cdot \frac{2}{x}\, dx$

$= \frac{1}{4}x^4 \ln(x^2) - \frac{1}{2}\int x^3\, dx$

$= \frac{1}{4}x^4 \ln(x^2) - \frac{1}{2}\frac{x^4}{4} + C$

$= \frac{1}{4}x^4 \ln(x^2) - \frac{1}{8}x^4 + C$

(b) *(5 marks)* $\int_1^2 \ln x\, dx$

LET $u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$= uv\Big|_1^2 - \int_1^2 v\, du$

$= x\ln x\Big|_1^2 - \int_1^2 x \cdot \frac{1}{x} dx$

$= x\ln x\Big|_1^2 - \int_1^2 dx$

$= x\ln x\Big|_1^2 - x\Big|_1^2 = 2\ln 2 - 1 \cdot \ln 1 - (2 - 1)$

$= 2\ln 2 - 1$

(c) *(5 marks)* $\int \frac{9x^3 - 18x^2 - x - 2}{3x+1}\,dx = I$

$$3x^2 - 7x + 2$$

$$3x+1 \,\overline{\big)\, 9x^3 - 18x^2 - x - 2}$$

$$-(9x^3 + 3x^2)$$

$$-21x^2 - x$$

$$-(-21x^2 - 7x)$$

$$6x - 2$$

$$-(6x + 2)$$

$$-4$$

$$\therefore\ I = \int 3x^2 - 7x + 2 - \frac{4}{3x+1}\,dx$$

$$= x^3 - \frac{7}{2}x^2 + 2x - \frac{4}{3}\ln|3x+1| + C$$

(d) *(5 marks)* $\int \frac{3x^2+8x+16}{x^3+8x^2+16x}\,dx = I$

$$x^3+8x^2+16x = x(x^2+8x+16) = x(x+4)(x+4) = x(x+4)^2$$

$$\frac{3x^2+8x+16}{x(x+4)^2} = \frac{A}{x} + \frac{B}{x+4} + \frac{C}{(x+4)^2}$$

$$3x^2+8x+16 = A(x+4)^2 + Bx(x+4) + Cx$$

IF $x=0$

$16 = 16A$

$1 = A$

IF $x=-4$

$48-32+16 = -4C$

$32 = -4C$

$\boxed{-8 = C}$

IF $x=1$

$3+8+16 = (1)(25) + B(1)(5) + (-8)(1)$

$27-25+8 = 5B$

$10 = 5B$

$\therefore \boxed{2 = B}$

$$\therefore I = \int \frac{1}{x} + \frac{2}{x+4} - \frac{8}{(x+4)^2}\,dx$$

$$\int \frac{8}{(x+4)^2}\,dx = 8\int u^{-2}\,du$$

$\boxed{\begin{array}{l}\text{LET } u = x+4 \\ du = dx\end{array}}$

$$= \frac{8u^{-1}}{-1} + c = -\frac{8}{(x+4)} + c$$

$$\therefore I = \ln|x| + 2\ln|x+4| - \frac{8}{(x+4)} + c$$

Question 3. *(5 marks)* Use the trapezoidal rule to approximate

$$\int_1^2 e^{1/x}\,dx$$

with $n = 4$. Round your final answer to 5 decimal places. (Marks will be deducted for rounding errors so keep enough decimal places while working.)

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$=1, x_1 = \frac{5}{4}, \quad x_2 = \frac{3}{2}, \quad x_3 = \frac{7}{4}, \quad x_4 = 2$$

$$\int_1^2 e^{1/x}\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$\approx \frac{1/4}{2}\left[e^{1/1} + 2e^{1/(5/4)} + 2e^{1/(3/2)} + 2e^{1/(7/4)} + e^{1/2}\right]$$

$$= \frac{1}{8}\left[2.718281828 + 4.451081857 + 3.895468082 + 3.541589905 + 1.648721271\right]$$

$$= \frac{1}{8}(16.25514294)$$

$$= 2.03189$$

Question 4. *(5 marks)* Use the table of integrals to find

$$\int \frac{dx}{e^x\sqrt{3-(e^x)^2}}$$

$$\boxed{\begin{aligned} \text{LET} \quad u &= e^x \\ du &= e^x\,dx \\ \frac{du}{e^x} &= dx \end{aligned}}$$

$$= \int \frac{1}{\underset{\underset{u}{\uparrow}}{e^x}\sqrt{3-u^2}} \, \frac{du}{\underset{\underset{u}{\uparrow}}{e^x}}$$

#21 FROM TABLE

$$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2 u} + c$$

so $a = \sqrt{3}$

$$= \int \frac{du}{u^2\sqrt{3-u^2}}$$

$$= -\frac{\sqrt{3-u^2}}{3u} + c$$

$$= -\frac{\sqrt{3-e^{2x}}}{3e^x} + c$$

Question 5. *(5 marks)* Evaluate the following limit

$$\lim_{x\to\infty} \frac{\ln(x+1)}{\sqrt{x}} \qquad \text{I.F. } \frac{\infty}{\infty}$$

$$\stackrel{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{\sqrt{x}}{2(x+1)} \qquad \text{I.F. } \frac{\infty}{\infty}$$

$$\stackrel{H}{=} \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{2} = \lim_{x\to\infty} \frac{1}{4\sqrt{x}} = 0$$

Question 6. [redacted] Evaluate the following improper integrals

(a) *(5 marks)* $\displaystyle\int_1^\infty \frac{1}{\sqrt{2x+1}}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{2x+1}}\,dx = I$

$$\int \frac{1}{\sqrt{2x+1}}\,dx = \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right) + C$$

$$\boxed{\begin{array}{l} \text{LET } u = 2x+1 \\ du = 2\,dx \\ \frac{du}{2} = dx \end{array}}$$

$$= \sqrt{2x+1} + C$$

$$\therefore \quad I = \lim_{b\to\infty}\Big[\sqrt{2x+1}\Big]_1^b = \lim_{b\to\infty}\left[\sqrt{2b+1} - \sqrt{2(1)+1}\right]$$

$$= \infty$$ (THE INTEGRAL DIVERGES)

(b) *(6 marks)* $\int_{-\infty}^{0} xe^x \, dx = \lim_{a \to -\infty} \int_a^0 xe^x dx = I$

$$\int xe^x dx = uv - \int v du$$

$\boxed{\begin{array}{ll} \text{LET } u = x & dv = e^x dx \\ du = dx & v = e^x \end{array}}$

$$= xe^x - \int e^x dx = xe^x - e^x + C$$

$$\therefore \quad I = \lim_{a \to -\infty} \left[xe^x - e^x \right]_a^0 = \lim_{a \to -\infty} \left[(0 - e^0) - (ae^a - e^a) \right]$$

$$= \lim_{a \to -\infty} \left[-1 - ae^a + e^a \right]$$

$$\lim_{a \to -\infty} ae^a = \lim_{a \to -\infty} \frac{a}{e^{-a}} \quad \text{I.F. } \frac{\infty}{\infty}$$

$$\overset{H}{=} \lim_{a \to -\infty} \frac{1}{-e^{-a}} = 0$$

$$\therefore \quad I = -1 - 0 + 0$$

$$= -1.$$

Bonus Question. *(2 marks)* The integral

$$\int_0^1 \frac{e^x}{e^x - 1}\,dx$$

is also called an improper integral. Why? How should we evaluate this ~~limit~~ INTEGRAL?? OOPS, GAVE IT AWAY.

IT IS IMPROPER BECAUSE $\dfrac{e^x}{e^x - 1}$ IS NOT DEFINED AT $x = 0$

WE SHOULD DEFINE

$$\int_0^1 \frac{e^x}{e^x - 1}\,dx = \lim_{a \to 0^+} \int_a^1 \frac{e^x}{e^x - 1}\,dx$$

TO EVALUATE THE INTEGRAL.