

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 7

Question 1. (3 marks) Without using a calculator, find the exact value of the following expression (Show your work.)

$$\begin{aligned}\log_2 6 - \log_2 15 + \log_2 20 &= \log_2 \left(\frac{6}{15} \right) + \log_2 (20) \\ &= \log_2 \left(\frac{6 \cdot 20}{15} \right) = \log_2 (8) = 3 \quad (\text{since } 2^3 = 8)\end{aligned}$$

Question 2. Find the derivatives of the following functions (you do not have to simplify your final answer).

(a) (3 marks) $y = 2x \log_{10} \sqrt{x}$

$$\begin{aligned}y' &= 2 \log_{10} \sqrt{x} + 2x \cdot \frac{d}{dx} (\log_{10} (x^{1/2})) \\ &= 2 \log_{10} \sqrt{x} + 2x \cdot \frac{1}{x^{1/2} \cdot \ln 10} \cdot \frac{d}{dx} (x^{1/2}) \\ &= 2 \log_{10} \sqrt{x} + 2x \cdot \frac{1}{x^{1/2} \ln 10} \cdot \frac{1}{2} x^{-1/2}\end{aligned}$$

(b) (4 marks) $g(y) = \ln \frac{(2y+1)^5}{\sqrt{y^2+1}}$

METHOD 1:

$$g'(y) = \left(\frac{1}{\frac{(2y+1)^5}{\sqrt{y^2+1}}} \right) \cdot \frac{d}{dy} \left(\frac{(2y+1)^5}{\sqrt{y^2+1}} \right)$$

$$= \frac{\sqrt{y^2+1}}{(2y+1)^5} \cdot \frac{5(2y+1)^4 \cdot 2 \cdot (y^2+1)^{1/2} - (2y+1)^5 \cdot \frac{1}{2}(y^2+1)^{-1/2}(2y)}{(\sqrt{y^2+1})^2}$$

METHOD 2:

$$g(y) = \ln(2y+1)^5 - \ln(y^2+1)^{1/2} = 5 \ln(2y+1) - \frac{1}{2} \ln(y^2+1)$$

$$\therefore g'(y) = 5 \left(\frac{1}{2y+1} \right) (2) - \frac{1}{2} \left(\frac{1}{y^2+1} \right) (2y)$$