Test 1

Question 1. *(5 marks)* Find the limit of the following sequence if it converges:

a) $\lim_{n\to\infty}\left(3+\frac{2}{n}\right)^n$ b) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n$

Question 2. *(5+5 marks)* Determine if the series is convergent or divergent. If it is convergent find its sum.

(a) $\sum_{n=2}^{\infty}\frac{3\cdot 6^n+4\cdot 6^{n+1}}{9^{n-1}}$

(b) $\sum_{n=1}^{\infty}[\arctan(n+1)-\arctan(n)]$

Question 3. *(3+5+5 marks)* Determine if the following series are convergent or divergent.

(a) $\sum_{k=1}^{\infty}\frac{(3k-2)(k^2-5)}{(k+1)(k^3+1)^2}$

(b) $\sum_{n=1}^{\infty}\frac{2^{n+1}(\ln n)^n}{n^n}$

(c) $\sum_{n=1}^{\infty}\frac{\ln n}{n^4}$

Question 4. *(6 marks)* Determine if the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1}$$

Question 5. *(6 marks)* Find the interval of convergence and radius of convergence of

$$\sum_{n=1}^{\infty}\frac{n}{5^n}(x+2)^n$$

Question 6. *(5+5+2marks)* Given that the function $f(x) = 1/(1+x^2)$ has the power series expansion

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + x^8 - \ldots$$

when x is in $(-1,1)$, find a power series expansion for the following. What can you say about the interval of convergence and the radius of convergence for each power series?

(a) $f(x) = \dfrac{1}{4+x^2}$

(b) $g(x) = \dfrac{1-x^2}{(1+x^2)^2}$

$$\left(\text{Hint: } \frac{1-x^2}{(1+x^2)^2} = \frac{d}{dx}\left(\frac{x}{1+x^2}\right) \text{ so first find a power series represntation for } \frac{x}{1+x^2}\right)$$

Bonus. Do one of the two following questions:

1) *(5 marks)* Determine whether or not the following series is convergent or divergent:

$$\sum_{n=3}^{\infty} [\ln(e \cdot n) - \ln(e \cdot n + 1)]$$

or

2) *(5 marks)* Prove that a number that has a deciamal expansion of the type

$$0.\overline{a_1 a_2 a_3 a_4 \ldots a_k} = 0.a_1 a_2 a_3 a_4 \ldots a_k a_1 a_2 a_3 a_4 \ldots a_k a_1 a_2 a_3 a_4 \ldots a_k \ldots$$

that is, has k repeating decimal places, can be writen as a ratio of integers (fraction).