

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #51a. (2 marks) Show that if A is invertible and $AB = AC$, then $B = C$

Premise (Given): A invertible $\Leftrightarrow AA^{-1} = I = A^{-1}A$
 $AB = AC$

$$\begin{aligned} AB &= AC \\ A^{-1}AB &= A^{-1}AC \\ IB &= IC \\ B &= C \end{aligned}$$

Question 2. §1.4 #53a. (3 marks) Show that if A , B , and $A + B$ are invertible matrices with the same size then

$$A(A^{-1} + B^{-1})B(A+B)^{-1} = I$$

Premise (Given):

A invertible	\Leftrightarrow	$AA^{-1} = I = A^{-1}A$
B "	\Leftrightarrow	$BB^{-1} = I = B^{-1}B$
$A+B$ "	\Leftrightarrow	$(A+B)(A+B)^{-1} = I = (A+B)^{-1}(A+B)$

$$\begin{aligned} \text{LHS} &= A(A^{-1} + B^{-1})B(A+B)^{-1} \\ &= (AA^{-1} + AB^{-1})B(A+B)^{-1} \\ &= (I + AB^{-1}B)(A+B)^{-1} \\ &= (B+A)(A+B)^{-1} \\ &= I = \text{RHS} \end{aligned}$$

Question 3. §1.4 #17 (5 marks) Use the given information to find A .

$$(I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$$

$$[(I + 2A)^{-1}]^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}^{-1}$$

$$I + 2A = \frac{1}{(-1)(5) - 4(2)} \begin{bmatrix} 5 & -2 \\ -4 & -1 \end{bmatrix}$$

$$I + 2A = \frac{-1}{13} \begin{bmatrix} 5 & -2 \\ -4 & -1 \end{bmatrix}$$

$$I + 2A = \begin{bmatrix} -5/13 & 2/13 \\ 4/13 & -1/13 \end{bmatrix}$$

$$2A = \begin{bmatrix} -5/13 & 2/13 \\ 4/13 & -1/13 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} -18/13 & 2/13 \\ 4/13 & -12/13 \end{bmatrix}$$

$$A = \frac{1}{2} \begin{bmatrix} -18/13 & 2/13 \\ 4/13 & -12/13 \end{bmatrix}$$

$$A = \begin{bmatrix} -9/13 & 1/13 \\ 2/13 & -6/13 \end{bmatrix}$$