Dawson College: **Linear Algebra**: 201-105-05-S3: Fall 2012

Name: Y. Lamontagne
Student ID: _____

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.5 #7d. *(2 marks)*

$$A=\begin{bmatrix}3 & 4 & 1\\ 2 & -7 & -1\\ 8 & 1 & 5\end{bmatrix} \quad C=\begin{bmatrix}3 & 4 & 1\\ 2 & -7 & -1\\ 2 & -7 & 3\end{bmatrix}$$

Find an elementary matrix E that satisfies the equation.

$EC=A$

$C \sim A$ (under $2R_1+R_3 \to R_3$)

$I \sim \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix} = E$ (under $2R_1+R_3 \to R_3$)

Question 2. §1.6 #2. *(3 marks)* Solve the system by inverting the coefficient matrix.

$$\begin{aligned} 4x_1 - 3x_2 &= -3\\ 2x_1 - 5x_2 &= 9 \end{aligned}$$

$Ax=b$

$x=A^{-1}b$

$$A=\begin{bmatrix}4 & -3\\ 2 & -5\end{bmatrix},\ b=\begin{bmatrix}-3\\ 9\end{bmatrix}$$

$$A^{-1}=\frac{1}{-20+6}\begin{bmatrix}-5 & 3\\ -2 & 4\end{bmatrix}=\begin{bmatrix}-5/-14 & 3/-14\\ -2/-14 & 4/-14\end{bmatrix}$$

$$x=\begin{bmatrix}+5/14 & -3/14\\ 1/7 & -2/7\end{bmatrix}\begin{bmatrix}-3\\ 9\end{bmatrix}=\begin{bmatrix}-42/14\\ -21/7\end{bmatrix}=\begin{bmatrix}-3\\ -3\end{bmatrix}$$

Question 3. §1.7 #17 *(2 marks)* Find A^2, A^{-2}.

$$A=\begin{bmatrix}-2 & 0 & 0 & 0\\ 0 & -4 & 0 & 0\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & 2\end{bmatrix}$$

$$A^{-2}=\begin{bmatrix}1/4 & 0 & 0 & 0\\ 0 & 1/16 & 0 & 0\\ 0 & 0 & 1/9 & 0\\ 0 & 0 & 0 & 1/4\end{bmatrix}$$

$$A^2=\begin{bmatrix}4 & 0 & 0 & 0\\ 0 & 16 & 0 & 0\\ 0 & 0 & 9 & 0\\ 0 & 0 & 0 & 4\end{bmatrix}$$

Question 4. §1.7 #17 *(3 marks)* Prove: If $A^TA=A$, then A is symmetric and $A=A^2$.

Premise: ① $A^TA=A$

Want to Show: ① $A^T=A$ ② $A^2=A$

① $LHS = A^T = [A^TA]^T = A^T[A^T]^T = A^TA = A = RHS$

② $LHS = A^2 = AA = A^TA = A = RHS$