Dawson College: Linear Algebra: 201-105-05-S3: Fall 2012

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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.1 #25 *(5 marks)* Evaluate $\det(A)$ by a cofactor expansion along a row or column of your choice.

$$A = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

$\det A = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33} + a_{43}C_{43}$

$= 0\,C_{13} + 0\,C_{23} - 3(-1)^{3+3}M_{33} + 3(-1)^{4+3}M_{43}$

$= -3\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} - 3\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}$

$= -3\left[a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}\right] - 3\left[a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}\right]$

$= -3\left[3(-1)^{1+1}\begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} + 3(-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5(-1)^{1+3}\begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix}\right]$

$-3\left[5(-1)^{1+3}\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 2(-1)^{2+3}\begin{vmatrix} 3 & 3 \\ 4 & 1 \end{vmatrix}\right]$

$= -3\left[3(4+20) - 3(4+4) + 5(20-4)\right] - 3\left[5(2-6) + 2(3-12)\right] = -240.$

Question 2. §2.2 #24 *(5 marks)* Evaluate the determinant of the given matrix by reducing the matrix to row echelon form.

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix} \sim \begin{matrix} -5R_1+R_2\to R_2 \\ R_1+R_3\to R_3 \\ -2R_1+R_4\to R_4 \end{matrix} \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & -1 \end{bmatrix} \sim$$

$$\begin{matrix} \\ \\ \\ -12R_2+R_4\to R_4 \end{matrix} \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 118 & +23 \end{bmatrix}$$

$$\sim \begin{matrix} \\ \\ \\ 36R_3+R_4\to R_4 \end{matrix} \begin{bmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & -13 \end{bmatrix} = B$$

$\det B = \det A$

$(1)(1)(-3)(-13) = \det A$

$\det A = 39$