Dawson College: Linear Algebra: 201-105-05-S3: Fall 2012

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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.3 #22 *(5 marks)* Decide whether the given matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$$

$|A| = 2(1)(6) = 12$

$C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 0 \\ 3 & 6 \end{vmatrix} = 6$

$C_{12} = (-1)^{1+2}\begin{vmatrix} 8 & 0 \\ -5 & 6 \end{vmatrix} = -48$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 8 & 1 \\ -5 & 3 \end{vmatrix} = 29$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 0 \\ 3 & 6 \end{vmatrix} = 0$

$C_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 0 \\ -5 & 6 \end{vmatrix} = 12$

$C_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 0 \\ -5 & 3 \end{vmatrix} = -6$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$C_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 0 \\ 8 & 0 \end{vmatrix} = 0$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 0 \\ 8 & 1 \end{vmatrix} = 2$

$\text{adj } A = [\text{matrix of cofactors}]^t$

$$= \begin{bmatrix} 6 & -48 & 29 \\ 0 & 12 & -6 \\ 0 & 0 & 2 \end{bmatrix}^t$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ -48 & 12 & 0 \\ 29 & -6 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{12}\begin{bmatrix} 6 & 0 & 0 \\ -48 & 12 & 0 \\ 29 & -6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ -4 & 1 & 0 \\ 29/12 & -1/2 & 1/6 \end{bmatrix}$$

Question 2. §2.3 #27 *(5 marks)* Solve by Cramer's rule.

$$\begin{aligned} x_1 - 3x_2 + x_3 &= 4 \\ 2x_1 - x_2 &= -2 \\ 4x_1 - 3x_3 &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{bmatrix}\begin{matrix} 1 & -3 \\ 2 & -1 \\ 4 & 0 \end{matrix} \quad b = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

$$|A_1| = \begin{vmatrix} 4 & -3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{vmatrix} = -3(-1)^{3+3}\begin{vmatrix} 4 & -3 \\ -2 & -1 \end{vmatrix} = -3(-10) = 30$$

$$|A_2| = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & -3 \end{vmatrix} = 4(-1)^{3+1}\begin{vmatrix} 4 & 1 \\ -2 & 0 \end{vmatrix} + (-3)(-1)^{3+3}\begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix}$$

$$= 4(2) + (-3)(-10) = 38$$

$$|A_3| = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix}$$

$$= 4(-1)^{3+1}\begin{vmatrix} -3 & 4 \\ -1 & -2 \end{vmatrix} = 40$$

$|A| = 1(-1)(-3) - (1)(-1)(4) - (-3)(2)(-3) = -11$

$\therefore x_1 = \frac{|A_1|}{|A|} = -\frac{30}{11}$, $x_2 = \frac{|A_2|}{|A|} = -\frac{38}{11}$

$x_3 = \frac{|A_3|}{|A|} = -\frac{40}{11}$