Dawson College: **Linear Algebra**: 201-105-05-S3: Fall 2012

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Quiz # 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.3 #5 *(2 marks)* Find a unit vector that is orthogonal to both $\mathbf{u} = (1,0,1)$ and $\mathbf{v} = (0,1,1)$.

$\vec{w} = \vec{u} \times \vec{v} = \left(\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\right) = (-1,-1,1)$

$1 \quad 0$
$0 \quad 1$
$1 \quad 1$

unit vector with same direction as $\vec{w}$ $= \frac{1}{\|\vec{w}\|}\vec{w} = \frac{1}{\sqrt{(-1)^2+(-1)^2+1^2}}(-1,-1,1) = \frac{1}{\sqrt{3}}(-1,-1,1) = \left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Question 2. §3.3 #26 *(3 marks)* Find the vector component of **u** along **a** and the vector component of **u** orthogonal to **a**.

$\vec{u} = (2,0,1)$, $\mathbf{a} = (1,2,3)$ $\quad \text{proj}_{\vec{a}}\vec{u} = \frac{\vec{u}\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{(2,0,1)\cdot(1,2,3)}{(1,2,3)\cdot(1,2,3)}(1,2,3) = \frac{2(1)+0(2)+1(3)}{1(1)+2(2)+3(3)}(1,2,3)$

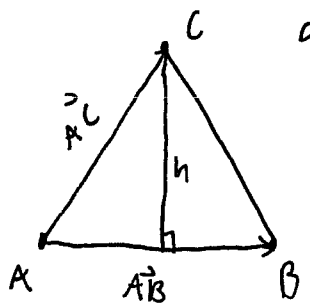
$= \frac{5}{14}(1,2,3) = \left(\frac{5}{14}, \frac{10}{14}, \frac{15}{14}\right)$

$u - \text{proj}_{\vec{a}}\vec{u} = (2,0,1) - \left(\frac{5}{14}, \frac{10}{14}, \frac{15}{14}\right) = \left(\frac{23}{14}, \frac{-10}{14}, \frac{-1}{14}\right)$

Question 3. §3.5 #27 *(5 marks)*

(a) *(3 marks)* Find the area of the triangle having vertices $A(1,0,1)$, $B(0,2,3)$, and $C(2,1,0)$.

(b) *(2 marks)* Use the result of part (a) to find the length of the altitude from vertex C to side AB.



a) $\vec{AB} = B - A = (0,2,3) - (1,0,1) = (-1,2,2)$
$\vec{AC} = C - A = (2,1,0) - (1,0,1) = (1,1,-1)$

$\|AB\| = \sqrt{(-1)^2+2^2+2^2} = \sqrt{9} = 3$

$\text{Area} = \frac{1}{2}\|\vec{AB} \times \vec{AC}\| = \frac{1}{2}\left\|\left(\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}, -\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix}\right)\right\| = \frac{1}{2}\|(-4,1,-3)\|$

$-1 \quad 1$
$2 \quad 1$
$2 \quad -1$

$= \frac{1}{2}\sqrt{(-4)^2+1^2+(-3)^2}$

$= \frac{1}{2}\sqrt{26}$

b) $A = \frac{1}{2}bh$

$\frac{1}{2}\sqrt{26} = \frac{1}{2}\|\vec{AB}\|\,h$

$h = \frac{\sqrt{26}}{\|\vec{AB}\|} = \frac{\sqrt{26}}{3}$