Name: _____

Student ID: _____

Test 1

This test is graded out of 55 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(10 marks)* Solve the following system by Gauss-Jordan elimination:

$$
\begin{array}{rcrcrcrcrcr}
3x_1 & - & x_2 & + & x_3 & - & 3x_4 & + & x_5 & = & 3 \\
4x_1 & + & 3x_2 & - & x_3 & + & x_4 & - & x_5 & = & 2 \\
7x_1 & + & 2x_2 & & & - & 2x_4 & + & 2x_5 & = & 1
\end{array}
$$

Question 2. Consider the matrices:

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 4 & -3 \\ 2 & -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & -4 & 0 \\ 2 & 2 & 1 \end{bmatrix} C = \begin{bmatrix} 0 & 1 \\ -3 & 2 \\ 0 & -1 \end{bmatrix} D = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} E = \begin{bmatrix} 2 & 0 & 0 \\ -4 & 0 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$E^{-1}$$

b. *(2 marks)* Compute the following, if possible.

$$D - BC$$

c. *(2 marks)* Compute the following, if possible.

$$tr(E)$$

d. *(3 marks)* Compute the following, if possible.

$$C^t B^t$$

e. *(3 marks)* Compute the following, if possible.

$$AB^t$$

f. *(5 marks)* Find F, if possible.

$$(F^{-1} + 2I)^{-1} = (BC)^t$$

Question 3. *(5 marks)* Solve the following system by Gaussian elimination:

$$
\begin{array}{rcrcrcrcl}
2x_1 & - & x_2 & - & x_3 & - & 3x_4 & = & 0 \\
3x_1 & + & 2x_2 & - & x_3 & + & x_4 & = & 1
\end{array}
$$

Question 4. *(6 marks)* Express

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

and A^{-1} as a product of elementary matrices. *Explicitely write all elementary matrices.*

Question 5. *(7 marks)* Solve the system by using and inverting the coefficient matrix.

$$
\begin{array}{rrrrrrcr}
x & - & 3y & + & z & = & 4 \\
2x & - & 5y & + & z & = & -1 \\
2x & - & 6y & + & 3z & = & 3
\end{array}
$$

Question 6. A square matrix A is said to be *skew-symmetric* if $A^T = -A$. Prove:

a. *(2 marks)* If A is an invertible skew-symmetric matrix, then A^{-1} is skew-symmetric.

b. *(3 marks)* If A and B are skew-symmetric matrices, then so are A^T, $A \pm B$, and kA for any scalar.

Question 7. *(5 marks)* Determine the conditions on the b_i's, if any, in order to guarantee that the linear system is consistent

$$
\begin{array}{rcrcrcl}
3x_1 & - & 2x_2 & + & 3x_3 & = & 1 \\
6x_1 & - & 6x_2 & + & 6x_3 & = & b_1 \\
-3x_1 & + & 2x_2 & - & 3x_3 & = & b_2
\end{array}
$$

Bonus Question. *(5 marks)* Prove that if $ad - bc \neq 0$, then the reduced row echelon form of

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$