Dawson College: **Linear Algebra**: 201-105-dw-S03: Fall 2012

Name: _____
Student ID: _____

Test 3

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given the following vertices $A(3,2)$, $B(5,4)$, $C(9,4)$, $D(7,2)$.

a. *(3 marks)* Find the area of the parallelogram *ABCD* using the determinant.

b. *(3 marks)* Find the length of the altitude from the side *AD* to side *BC* using projections.

c. *(1 mark)* Find the area of the parallelogram *ABCD* using part b.

Question 2. *(2 marks)* Given

$$\begin{array}{lrcll}
\mathscr{L}_1: & (x,y,z) & = & (2+5t,1+t,-t) & t\in\mathbb{R}\\
\mathscr{L}_2: & (x,y,z) & = & (7+2t,4,10t) & t\in\mathbb{R}\\
\mathscr{L}_3: & (x,y,z) & = & (9-t,2,9-5t) & t\in\mathbb{R}\\
\mathscr{P}_1: & x-2y+3z-11 & = & 0 & \\
\mathscr{P}_2: & -5x-y+z+31 & = & 0 & \\
\mathscr{P}_3: & -3x+6y-9z+1 & = & 0 &
\end{array}$$

a. *(2 marks)* Find a vector parallel to the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$.

b. *(2 marks)* Find a unit vector perpendicular to $\mathscr{P}_3$.

c. *(1 marks)* Find a vector parallel to $\mathscr{L}_3$.

d. *(3 marks)* Find the point intersection between $\mathscr{P}_2$ and $\mathscr{L}_1$ if it exists.

Question 3. *(4 marks)* Find the distance between $A(1,0,3)$ and the plane $2x-y+3z+3=0$. **Do NOT use the formula:** $D=\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$

Question 4. *(6 marks)* Determine whether the line $\mathscr{L}_1$ passing through the point $A(2,3,3)$ and $B(3,1,8)$ intersects $\mathscr{L}_2: \quad (x,y,z)=(7+2t,4,10t) \quad t\in\mathbb{R}$.

Question 5. *(5 marks)* Given

$$\begin{array}{lrcl} \mathscr{P}_1\colon & -x-3y+2z-3 & = & 0 \\ \mathscr{P}_2\colon & 2x+4y-3z+5 & = & 0, \end{array}$$

find the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$.

Question 6. *(5 marks)* Using projections find the distance between the following parallel lines

$$\begin{array}{lrcll} \mathscr{L}_1\colon & (x,y,z) & = & (2+5t,1+t,-t) & t\in\mathbb{R} \\ \mathscr{L}_2\colon & (x,y,z) & = & (3-10t,4-2t,1+2t) & t\in\mathbb{R}. \end{array}$$

Question 7. *(5 marks)* Find the equation of the plane containing the line $(x,y,z) = (2+2t, 4-t, 1-3t) \quad t \in \mathbb{R}$ and the point $A(1,0,-3)$.

Question 8. *(5 marks)* Maximize $Z = 3x + y$ subject to $2x - y \leq 60$, $x + y \leq 50$.

Question 9. *(5 marks)* Minimize $Z = x + y$ subject to $x + y \geq 2$, $3x + y \geq 4$.

Bonus Question. *(5 marks)* Prove: If θ is the angle between $\mathbf{u}$ and $\mathbf{v}$ then $||\mathbf{u} \times \mathbf{v}|| = ||\mathbf{u}||||\mathbf{v}|| \sin\theta$. *Hint: Use Lagrange's identity:* $||\mathbf{u} \times \mathbf{v}||^2 = ||\mathbf{u}||^2||\mathbf{v}||^2 - (\mathbf{u} \cdot \mathbf{v})^2$.