

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (10 marks) Maximize $Z = 2x - y + 4z + 2w$ subject to $y + z + w \leq 30$, $2x - y + 2z - w \leq 60$, $2x - y - z - w \leq 10$

Convert to equations

$$\begin{aligned} y + z + w + S_1 &= 30 \\ 2x - y + 2z - w + S_2 &= 60 \\ 2x - y - z - w + S_3 &= 10 \\ -2x + y - 4z - 2w + Z &= 0 \end{aligned}$$

$$\begin{aligned} &\begin{array}{c} \swarrow \text{pivot} \\ \left[\begin{array}{cccc|cccc|c} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 30 \\ 2 & -1 & 2 & -1 & 0 & 1 & 0 & 0 & 60 \\ 2 & -1 & -1 & -1 & 0 & 0 & 1 & 0 & 10 \\ -2 & 1 & -4 & -2 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} r = \frac{30}{1} = 30 \leftarrow \text{p.r.} \\ r = \frac{60}{2} = 30 \\ r = \frac{10}{2} = 5 \end{array} \\ \uparrow \text{p.c.} \end{array} \rightarrow \begin{array}{c} \swarrow \text{pivot} \\ \left[\begin{array}{cccc|cccc|c} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 30 \\ 0 & -3 & 0 & -3 & -2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 40 \\ -2 & 5 & 0 & 2 & 4 & 0 & 0 & 1 & 120 \end{array} \right] \begin{array}{l} r = \frac{0}{-3} = 0 \\ r = \frac{40}{2} = 20 \\ r = \frac{120}{-2} = -60 \end{array} \\ \uparrow \text{p.c.} \end{array} \end{aligned}$$

$$\begin{aligned} &\begin{array}{c} \swarrow \text{pivot} \\ \left[\begin{array}{cccc|cccc|c} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 30 \\ 1 & -3/2 & 0 & -3/2 & -1 & 1/2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 40 \\ -2 & 5 & 0 & 2 & 4 & 0 & 0 & 1 & 120 \end{array} \right] \begin{array}{l} r = \frac{30}{1} = 30 \\ r = \frac{40}{3} \leftarrow \text{p.r.} \end{array} \\ \uparrow \text{p.c.} \end{array} \rightarrow \begin{array}{c} \swarrow \text{pivot} \\ \left[\begin{array}{cccc|cccc|c} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 30 \\ 1 & -3/2 & 0 & -3/2 & -1 & 1/2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 3 & -1 & 1 & 0 & 40 \\ 0 & 2 & 0 & -1 & 2 & 1 & 0 & 1 & 120 \end{array} \right] \begin{array}{l} r = \frac{30}{1} = 30 \\ r = \frac{40}{3} \leftarrow \text{p.r.} \end{array} \\ \uparrow \text{p.c.} \end{array} \end{aligned}$$

$$\begin{aligned} &\begin{array}{c} \swarrow \text{pivot} \\ \left[\begin{array}{cccc|cccc|c} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 30 \\ 1 & -3/2 & 0 & -3/2 & -1 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1/3 & 1/3 & 1 & 0 & 40/3 \\ 0 & 2 & 0 & -1 & 2 & 1 & 0 & 0 & 120 \end{array} \right] \begin{array}{l} r = \frac{30}{1} = 30 \\ r = \frac{40}{3} \leftarrow \text{p.r.} \end{array} \\ \uparrow \text{p.c.} \end{array} \rightarrow \begin{array}{c} \left[\begin{array}{cccc|cccc|c} 0 & 0 & 1 & 0 & 0 & 1/3 & -1/3 & 0 & 50/3 \\ 1 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 & 20 \\ 0 & 1 & 0 & 1 & 1 & -1/3 & 1/3 & 0 & 40/3 \\ 0 & 3 & 0 & 0 & 3 & 2/3 & 1/3 & 1 & 400/3 \end{array} \right] \begin{array}{l} -R_3 + R_1 \rightarrow R_1 \\ \frac{3}{2}R_3 + R_2 \rightarrow R_2 \\ R_3 + R_4 \rightarrow R_4 \end{array} \end{array} \end{aligned}$$

$x = 20, y = 0, z = \frac{50}{3}, w = \frac{40}{3}, S_1 = 0, S_2 = 0, S_3 = 0, Z = \frac{400}{3}$