

## Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1. (10 marks)** Minimize  $Z = 3x + y - 2z$  subject to  $2x - y \geq 60$ ,  $x + y - 2z \geq 50$ ,  $x - y - z \geq 10$ .

Change to maximizing prob.  $P = -Z = -3x - y + 2z$  and change inequalities to equalities

$$\begin{array}{rcl} 2x - y & -s_1 & = 60 \\ x + y - 2z & -s_2 & = 50 \\ x - y - z & -s_3 & = 10 \\ 3x + y - 2z & +P & = 0 \end{array}$$

Initial tableau:

$$\begin{array}{c} \# \\ \# \\ * \end{array} \left[ \begin{array}{cccccc|c} 2 & -1 & 0 & -1 & 0 & 0 & 60 \\ 1 & 1 & -2 & 0 & -1 & 0 & 50 \\ 1 & -1 & -1 & 0 & 0 & -1 & 10 \\ 3 & 1 & -2 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} r = \frac{60}{2} = 30 \\ r = \frac{50}{1} = 50 \\ r = \frac{10}{1} = 10 \leftarrow \text{P.V.} \end{array}$$

Row operations:

$$\begin{array}{l} -2R_3 + R_1 \rightarrow R_1 \\ -R_3 + R_2 \rightarrow R_2 \\ -3R_3 + R_4 \rightarrow R_4 \end{array} \left[ \begin{array}{cccccc|c} 0 & 1 & 2 & -1 & 0 & 2 & 40 \\ 0 & 2 & -1 & 0 & -1 & 1 & 40 \\ 1 & -1 & -1 & 0 & 0 & -1 & 10 \\ 0 & 4 & 1 & 0 & 0 & 3 & -30 \end{array} \right] \leftarrow \text{P.V.}$$

Row operations:

$$\frac{1}{2}R_1 \left[ \begin{array}{cccccc|c} 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 1 & 20 \\ 0 & 2 & -1 & 0 & -1 & 0 & 40 \\ 1 & -1 & -1 & 0 & 0 & -1 & 10 \\ 0 & 4 & 1 & 0 & 0 & 3 & -30 \end{array} \right]$$

Row operations:

$$\begin{array}{l} R_1 + R_2 \rightarrow R_2 \\ R_1 + R_3 \rightarrow R_3 \\ -R_1 + R_4 \rightarrow R_4 \end{array} \left[ \begin{array}{cccccc|c} 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 1 & 20 \\ 0 & \frac{5}{2} & 0 & -\frac{1}{2} & -1 & 2 & 60 \\ 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 30 \\ 0 & \frac{7}{2} & 0 & \frac{1}{2} & 0 & 2 & -50 \end{array} \right] \begin{array}{l} r = \frac{20}{\frac{1}{2}} = 40 \\ r = \frac{60}{\frac{5}{2}} = 24 \leftarrow \text{P.V.} \\ r = \frac{30}{1} = 30 \end{array}$$

Row operations:

$$\frac{2}{5}R_2 \left[ \begin{array}{cccccc|c} 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 1 & 20 \\ 0 & 1 & 0 & -\frac{1}{5} & -\frac{2}{5} & \frac{4}{5} & 24 \\ 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 30 \\ 0 & \frac{7}{2} & 0 & \frac{1}{2} & 0 & 2 & -50 \end{array} \right] \leftarrow \text{P.V.}$$

Row operations:

$$\begin{array}{l} -\frac{1}{2}R_2 + R_1 \rightarrow R_1 \\ \frac{1}{2}R_2 + R_3 \rightarrow R_3 \\ -\frac{7}{2}R_2 + R_4 \rightarrow R_4 \end{array} \left[ \begin{array}{cccccc|c} 0 & 0 & 1 & -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} & 8 \\ 0 & 1 & 0 & -\frac{1}{5} & -\frac{2}{5} & \frac{4}{5} & 24 \\ 1 & 0 & 0 & -\frac{3}{5} & -\frac{1}{5} & \frac{2}{5} & 42 \\ 0 & 0 & 0 & \frac{6}{5} & \frac{7}{5} & -\frac{4}{5} & -134 \end{array} \right]$$

Row ratios:

$$\begin{array}{l} r_1 = \frac{8}{\frac{3}{5}} = \frac{40}{3} \leftarrow \text{P.V.} \\ r_2 = \frac{24}{\frac{4}{5}} = 30 \\ r_3 = \frac{42}{\frac{2}{5}} = 105 \end{array}$$

Row operations:

$$\frac{5}{3}R_1 \left[ \begin{array}{cccccc|c} 0 & 0 & \frac{5}{3} & -\frac{2}{3} & \frac{1}{3} & 1 & \frac{40}{3} \\ 0 & 1 & 0 & -\frac{1}{5} & -\frac{2}{5} & \frac{4}{5} & 24 \\ 1 & 0 & 0 & -\frac{3}{5} & -\frac{1}{5} & \frac{2}{5} & 42 \\ 0 & 0 & 0 & \frac{6}{5} & \frac{7}{5} & -\frac{4}{5} & -134 \end{array} \right]$$

Row operations:

$$\begin{array}{l} -\frac{4}{5}R_1 + R_2 \rightarrow R_2 \\ -\frac{2}{5}R_1 + R_3 \rightarrow R_3 \\ \frac{4}{5}R_1 + R_4 \rightarrow R_4 \end{array} \left[ \begin{array}{cccccc|c} 0 & 0 & \frac{5}{3} & -\frac{2}{3} & \frac{1}{3} & 1 & \frac{40}{3} \\ 0 & 1 & -\frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 0 & \frac{40}{3} \\ 1 & 0 & -\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & \frac{110}{3} \\ 0 & 0 & \frac{4}{3} & \frac{2}{3} & \frac{5}{3} & 0 & -\frac{370}{3} \end{array} \right]$$

Final solution:

$$x = \frac{110}{3}, y = \frac{40}{3}, z = 0, s_1 = 0 = s_2, s_3 = \frac{40}{3}$$

Objective function value:

$$Z = -P = \frac{370}{3}$$