

Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #21 (10 marks) Solve the given linear system by any method.

$$\begin{aligned} 2x + 2y + 4z &= 0 \\ w - y - 3z &= 0 \\ 2w + 3x + y + z &= 0 \\ -2w + x + 3y - 2z &= 0 \end{aligned}$$

$$\begin{bmatrix} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix}$$

$$\sim R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{bmatrix}$$

$$\sim \begin{aligned} &\frac{1}{2}R_2 \\ &-2R_1 + R_3 \rightarrow R_3 \\ &2R_1 + R_4 \rightarrow R_4 \end{aligned} \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 3 & 3 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{bmatrix}$$

$$\sim \begin{aligned} &-3R_2 + R_3 \rightarrow R_3 \\ &-R_2 + R_4 \rightarrow R_4 \end{aligned} \begin{bmatrix} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 10 & 0 \end{bmatrix}$$

$$\sim \begin{aligned} &3R_3 + R_1 \rightarrow R_1 \\ &-2R_3 + R_2 \rightarrow R_2 \\ &-10R_3 + R_4 \rightarrow R_4 \end{aligned} \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $y = t$ and sub. into

$$\begin{cases} w - y = 0 \\ x + y = 0 \\ z = 0 \end{cases}$$

$$\begin{aligned} w &= t \\ x &= -t \\ y &= t \\ z &= 0 \end{aligned}$$

∴ $(w, x, y, z) = (t, -t, t, 0)$.

$\forall t \in \mathbb{R}$