

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §1.6 # 9 Solve the linear systems together by reducing the appropriate augmented matrix.

$$\begin{aligned} x_1 - 5x_2 &= b_1 \\ 3x_1 + 2x_2 &= b_2 \end{aligned}$$

$$(i) b_1 = 1, b_2 = 4 \quad (ii) b_1 = -2, b_2 = 5$$

$$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 3 & 2 & 4 & 5 \end{array} \right]$$

$$\sim -3R_1 + R_2 \rightarrow R_2 \left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 17 & 1 & 11 \end{array} \right]$$

$$\sim \frac{1}{17} R_2 \left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 1 & \frac{1}{17} & \frac{11}{17} \end{array} \right]$$

$$\sim 5R_2 + R_1 \rightarrow R_1 \left[\begin{array}{cc|c|c} 1 & 0 & \frac{22}{17} & \frac{21}{17} \\ 0 & 1 & \frac{1}{17} & \frac{11}{17} \end{array} \right]$$

$$i) (x_1, x_2) = \left(\frac{22}{17}, \frac{1}{17} \right)$$

$$ii) (x_1, x_2) = \left(\frac{21}{17}, \frac{11}{17} \right)$$

Question 2. §1.7 # 32 Let A be an $n \times n$ symmetric matrix.

a. (2 marks) Show that A^2 is symmetric.

b. (3 marks) Show that $2A^2 - 3A + I$ is symmetric.

a) Need to show $(A^2)^t = A^2$ with the premise $A^t = A$. LHS = $(A^2)^t = (A^t)^2 = (A)^2 = A^2 = \text{RHS}$

b) Need to show $2A^2 - 3A + I$ with the premise $A^t = A$.

$$\begin{aligned} \text{LHS} &= (2A^2 - 3A + I)^t = (2A^2)^t + (-3A)^t + I^t \\ &= 2(A^2)^t - 3A^t + I = 2A^2 - 3A + I = \text{RHS} \end{aligned}$$