

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) §2.1 # 15 Find all values of λ for which $\det(A) = 0$.

$$A = \begin{bmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{bmatrix} \quad \begin{aligned} 0 &= |A| = (\lambda - 2)(\lambda + 4) + 5 \\ 0 &= \lambda^2 + 2\lambda - 3 \\ 0 &= (\lambda + 3)(\lambda - 1) \\ \lambda &= -3 \quad \lambda = 1 \end{aligned}$$

Question 2. (5 marks) §2.1 # 25 Evaluate $\det(A)$ by a cofactor expansion along a row or column of your choice.

$$\begin{aligned} A &= \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix} & \det A &= a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33} + a_{43}C_{43} \\ & & &= 0C_{13} + 0C_{23} - 3(-1)^{3+3}M_{33} + (3)(-1)^{4+3}M_{43} \\ & & &= -3 \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix} \\ & & &= -3 [a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}] - 3 [a_{15}C_{15} + a_{23}C_{23} + a_{33}C_{33}] \\ & & &= -3 \left[3(-1)^{1+1} \begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} + 3(-1)^{1+2} \begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5(-1)^{1+3} \begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix} \right] \\ & & & - 3 \left[5(-1)^{1+3} \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 2(-1)^{2+3} \begin{vmatrix} 3 & 3 \\ 4 & 1 \end{vmatrix} \right] \\ & & &= -3 [3(4+20) - 3(4+4) + 5(20-4)] - 3[5(2-8) + 2(3-12)] \\ & & &= -3 [3(24) - 3(8) + 5(16)] - 3[5(-6) + 2(-9)] \\ & & &= -3 [72 - 24 + 80] - 3[-30 - 18] = -240 \end{aligned}$$

Question 3. (2 marks) §2.1 # 32 Evaluate the determinant of the given matrix by inspection.

$$\begin{bmatrix} -3 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 40 & 10 & -1 & 0 \\ 100 & 200 & -23 & 3 \end{bmatrix} = A \quad \det(A) = -3(2)(-1)(3) = 18$$