

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.2 #27 (5 marks) Evaluate the determinant

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$$

given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$$

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\sim \begin{matrix} -3R_1 \\ -4R_2 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{bmatrix} = B$$

$$\begin{aligned} \det B &= (-3) \det A \\ &= -3(-6) \\ &= 18 \end{aligned}$$

Question 2. §2.3 #24 (3 marks) Solve by Cramer's rule

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 3x_1 + x_2 &= 5 \end{aligned}$$

$$|A| = 7 + 6 = 13, \quad |A_1| = 26$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{26}{13} = 2$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{26}{13} = 2$$

$$\text{where } Ax = b, \quad \text{where } A = \begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$$

Question 3. §2.3 #39 (2 marks) Show that if A is a square matrix, then $\det(A^T A) = \det(AA^T)$.

$$\text{LHS} = \det(A^T A) = \det A^T \det A = \det A \det A^T = \det(AA^T) = \text{RHS}$$

