

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.2 #23d (5 marks) Find the cosine of the angle θ between $\mathbf{u} = (-2, 2, 3)$ and $\mathbf{v} = (1, 7, -4)$.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

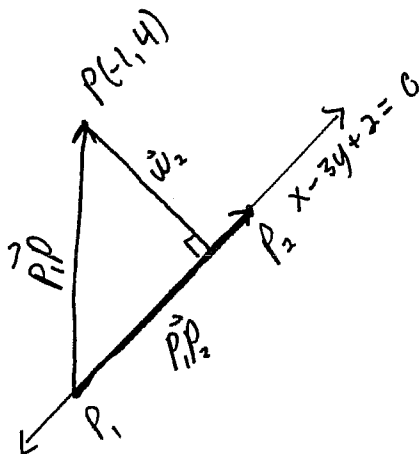
$$= \frac{(-2, 2, 3) \cdot (1, 7, -4)}{\sqrt{(-2)^2 + 2^2 + 3^2} \sqrt{1^2 + 7^2 + (-4)^2}}$$

$$= \frac{-2(1) + 2(7) + 3(-4)}{\sqrt{17} \sqrt{66}}$$

$$= \frac{0}{\sqrt{17} \sqrt{66}} = 0$$

Question 2. §3.3 #30 (5 marks) Find the distance between the point and the line. *YOU MUST use projections to solve this problem*

$(-1, 4); x - 3y + 2 = 0$



Let $y=0$
 $x - 3(0) + 2 = 0$
 $x = -2$

$P_1(-2, 0)$

Let $y=1$
 $x - 3(1) + 2 = 0$
 $x = 1$ $P_2(1, 1)$

$$\vec{P_1P_2} = P_2 - P_1 = (1, 1) - (-2, 0) = (3, 1)$$

$$P_1\vec{P} = P - P_1 = (-1, 4) - (-2, 0) = (1, 4)$$

$$\vec{w}_2 = \vec{P_1P} - \text{proj}_{\vec{P_1P_2}} \vec{P_1P}$$

$$= (1, 4) - \frac{P_1\vec{P} \cdot \vec{P_1P_2}}{P_1\vec{P_2} \cdot P_1\vec{P_2}} P_1\vec{P_2}$$

$$= (1, 4) - \frac{(3, 1) \cdot (1, 4)}{(3, 1) \cdot (3, 1)} (3, 1)$$

$$= (1, 4) - \frac{3+4}{9+1} (3, 1)$$

$$= (1, 4) - \left(\frac{21}{10}, \frac{7}{10}\right)$$

$$= \left(-\frac{11}{10}, \frac{33}{10}\right)$$

$$d = \|\vec{w}_2\|$$

$$= \sqrt{\left(-\frac{11}{10}\right)^2 + \left(\frac{33}{10}\right)^2}$$

$$= \sqrt{\frac{11^2 + 33^2}{10^2}}$$

$$= \frac{\sqrt{121 + 9(121)}}{10}$$

$$= \frac{\sqrt{10(121)}}{10}$$

$$= \frac{11}{\sqrt{10}}$$